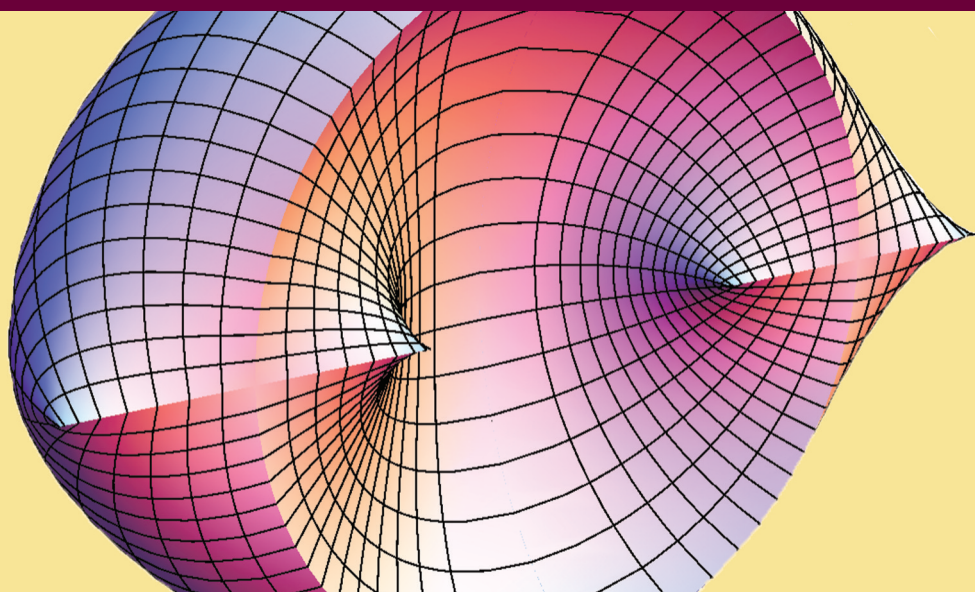


MECHANICAL ENGINEERING AND SOLID MECHANICS SERIES



Nonlinear Physical Systems

*Spectral Analysis, Stability
and Bifurcations*

Edited by
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ISTE

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