Chapter 11

Non-Hermitian perturbation of Hermitian matrices with physical applications

By the various sections of this solid, in several positions . . . divers new lines must arise, in a great variety, different from those arising from the section of a cone. Some of which . . . might be of good use in the building of ships.

John Wallis, *Cono-cuneus* [591]

In 1662 Peter Pett (1610–1672), Commissioner of the Navy and son of a King's Master Shipwright, in an attempt to produce ship hulls of least resistance, visited a famous Oxford mathematician John Wallis[1] (1616–1703). Wallis was asked for help with respect to a specific geometric solid that Pett intended to use in shaping a new shipwright's tool for laying off vessels [402]. The so-called *shipwright's circular wedge* was thought to consist of progressively curved strips of wood, which could then be disassembled, much like a half hull. What mathematical law should determine the variation of the curvature?

Wallis proposed to model ship hulls using a universal ruled surface generated by a line (imagine a plank of wood) moving on two directors, one of which is rectilinear and perpendicular to all generators (imagine a bow of a ship as a simple upright line meeting a level keel at the bottom in a right angle), and the other (situated midship) is a circle perpendicular to the plane which contains its center and the other director, Figure 11.1 (left). Extending the basic conoid shape through the vertical (bow) line into its reflection [402], Wallis found the equation of the surface to be

$$c^2x^2 = y^2(a^2 - z^2), \tag{11.1}$$

where a is the radius of the circular director and c the distance of its center on the y-axis from the origin in the (x, y, z)-space. The rectilinear director on the z-axis, and the line at infinity perpendicular to the latter are self-intersections of the surface, Figure 11.1 (right). In the 21st century, 'a great variety of lines' in the cross-sections of the *conical wedge of Wallis*[2] determine peculiar crossings and avoided crossings of eigenvalues in non-Hermitian problems of fluid dynamics [246, 458], optics [52, 154], and atomic physics [260, 365, 426], to name a few.

[1] We recall for example the *Wallis product* (1655): $\frac{2}{1}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\frac{4}{5}\cdot\frac{6}{5}\cdots = \frac{\pi}{2}$.

[2] The coffee-shop culture of our days quickly associated it with the 'double'-coffee-filter [260].