Chapter 3

Hamiltonian and gyroscopic systems

> *... for a uniformly travelling periodic water wave to lose spectral stability, it is necessary that there be for the linearized problem about it a collision of eigenvalues of opposite signature or at zero ... the signature of an eigenvalue ... is negative if ... the corresponding disturbance moves in the same direction as the wave but slower, and positive if it moves faster or in the opposite direction. If the signatures are the same we predict an avoided crossing; if they are opposite we predict a bubble of instability.*
>
> R. S. MacKay and P. G. Saffman [392]

An attempt to spin a hard-boiled egg always ends up successfully: when spun sufficiently rapidly, its symmetry axis can even rise to the vertical position demonstrating a gyroscopic stabilization. The mathematical model of this effect is the rotating solid prolate spheroid known as *Jellett's egg* [83, 290, 425]. In contrast, trying to spin a raw egg containing a yolk inside, surrounded by a liquid, will generally lead to its slow wobbling motion. Remarkably, already in 1877 Kelvin experimentally demonstrated that a thin-walled and slightly oblate spheroid completely filled with liquid remains stable if rotated fast enough about a fixed point, which does not happen if the spheroid is slightly prolate [572]. In 1880 this observation was confirmed theoretically by Greenhill [205], who found that rotation around the center of gravity of the top in the form of a weightless ellipsoidal shell completely filled with an ideal and incompressible fluid, is unstable when $a < c < 3a$, where c is the length of the semiaxis of the ellipsoid along the axis of rotation and the lengths of the two other semiaxes are equal to a [549].

Quite similarly, bullets and projectiles fired from barreled weapons can relatively easily be stabilized by rotation, if they are solid inside. In 1942 when the Second World War in Russia was in full swing and thus using even chemical weapons did not seem impossible, researchers at the Steklov Mathematical Institute (then evacuated to the city of Kazan, far to the east of Moscow) had to resolve a problem of instability of chemical artillery shells.[1] These shells, containing a liquid substance inside, had a

[1] According to Boris Isaakovich Rabinovich (1924–2010) – a Russian aerospace engineer who worked on dynamics and stability of liquid-fuel rockets and spacecrafts [488], see also [451].