Chapter 6

Dissipation-induced instabilities

Unfortunately, it is quite common for an eigenvalue which is moving steadily towards a positive growth rate to suffer a sudden change of direction and subsequently fail to become unstable; similarly, it happens that modes which initially become more stable as [the Reynolds number] increases change direction and subsequently achieve instability. It is believed that these changes of direction are due to the nearby presence of multiple-eigenvalue points.

C. A. Jones [246]

6.1 Crandall's gyropendulum

Crandall's gyropendulum [139] is an axisymmetric rigid body pivoted at a point O on the symmetry axis as shown in Figure 6.1 (a). When the axial spin Ω is absent, the upright position is statically unstable. When $\Omega \neq 0$, the body becomes a *gyroscopic pendulum*. Its primary parameters are its mass m, the distance L between the mass center and the pivot point, the axial moment of inertia I_a, and the diametral moment of inertia I_d about the pivot point. The gravity acceleration is denoted by g.

It is assumed that a drag force proportional to the linear velocity of the center of mass of the gyropendulum acts at the center of mass to oppose that velocity (stationary damping with the coefficient b_s). Additionally, it is assumed that a rigid sphere concentric with the pendulum tip O, is attached to the pendulum and rubs against a fixed rub plate. The gyropendulum is supported without friction at O, while a viscous friction force acts between the larger sphere and the rub plate, being responsible for the rotating damping with the coefficient b_r. The linearized equations of motion for the gyropendulum in the vicinity of the vertical equilibrium position derived in [139] have the form of the general nonconservative system (5.1) with the matrices $\mathbf{G}$, $\mathbf{D}$, $\mathbf{K}$, and $\mathbf{N}$ specified by the expressions

$$\mathbf{G} = \begin{pmatrix} 0 & \eta\Omega \\ -\eta\Omega & 0 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} \sigma+\rho & 0 \\ 0 & \sigma+\rho \end{pmatrix},$$
$$\mathbf{K} = \begin{pmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{pmatrix}, \quad \mathbf{N} = \begin{pmatrix} 0 & \rho\Omega \\ -\rho\Omega & 0 \end{pmatrix}. \tag{6.1}$$