Chapter 7

Nonself-adjoint boundary eigenvalue problems for differential operators and operator matrices dependent on parameters

Normally, the boundary conditions have great influence on the stability limits.

P. Pedersen [474]

Nonself-adjoint boundary eigenvalue problems for matrix differential operators describe distributed nonconservative systems with the coupled modes and appear in structural mechanics, fluid dynamics, magnetohydrodynamics, to name a few.

Practical needs for optimization and rational experiment planning in modern applications allow both the differential expression and the boundary conditions to depend analytically on the spectral parameter and smoothly on several physical parameters (which can be scalar or distributed) [85, 105, 106, 195, 307, 474].

According to the ideas going back to von Neumann and Wigner [587], in the multiparameter operator families, eigenvalues with various algebraic and geometric multiplicities can be generic [18, 130, 569]. In some applications additional symmetries [146, 147] yield the existence of *spectral meshes* [208] of eigencurves [63, 64] in the plane 'eigenvalue versus parameter' containing an infinite number of nodes with multiple eigenvalues. A classical example is the Campbell diagrams in rotor dynamics [109], see also [123, 245, 370, 583] and references therein.

As has been pointed out already by Rellich [495], sensitivity analysis of multiple eigenvalues is complicated by their nondifferentiability as functions of several parameters. Singularities corresponding to multiple eigenvalues [18] are related to such important effects as the destabilization paradox in near-Hamiltonian and near-reversible systems [81, 90, 232, 312, 333, 354], geometric phase [154, 397], transfer of instability between modes in dissipative problems of fluid mechanics [106, 246, 458], reversals of the geomagnetic field [195, 545], emission of sound by rotating continua interacting with the friction pads [123, 254, 294, 538, 604], material instabilities (flutter) in elastoplastic flows [59, 60], and other phenomena [518].

An increasing number of multiparameter nonself-adjoint boundary eigenvalue problems and the need for simple constructive estimates of critical parameters and eigenvalues as well as for verification of numerical codes and discretization schemes [75, 107, 249, 419] require development of applicable methods, allowing one to track relatively easily and conveniently the changes in simple and multiple eigenvalues and the corresponding eigenvectors due to variation of the differential expression and es-