Chapter 9

The MHD kinematic mean field α^2-dynamo

Although eigencurves have been used in a variety of ways for about a century, they seem comparatively underdeveloped in their own right.

P. Binding and H. Volkmer [64]

Motions of an electrically conducting medium (plasma or liquid metal) can produce a magnetic field. Electric currents and fields that are generated by the moving fluid amplify the existing magnetic field and maintain it against resistive decay. The process of conversion of mechanical energy into magnetic energy is called *dynamo*. The magnetohydrodynamics (MHD) dynamo is the main mechanism that creates magnetic fields of stars and planets [199, 359].

9.1 Eigenvalue problem for α^2-dynamo

The motion of a conducting fluid in a magnetic field is described by the MHD equations that basically couple the Navier–Stokes equations of a viscous fluid to the induction equation for a magnetic field [199]. This set of nonlinear partial differential equations with boundary conditions requires as a rule computationally expensive numerical simulations [197, 198]. Nevertheless, the dynamo action can be understood already by means of simplified models of the *kinematic dynamo* theory based on the assumption that the velocity field of the fluid, $\mathbf{v}$, is prescribed and not affected by the magnetic field, $\mathbf{B}$.

In the kinematic regime, the time evolution of the magnetic field is reduced to the linear induction equation

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{9.1}$$

where $\nabla \cdot \mathbf{B} = 0$ and $\eta = (\mu_0 \sigma)^{-1}$ is the magnetic diffusivity, $\sigma = \text{const}$ the electrical conductivity of the fluid, and μ_0 the magnetic permeability of free space. Although a number of flows are known that make the kinematic dynamo (9.1) work, see e.g. [188], it is a hard problem to find such a flow configuration [247].