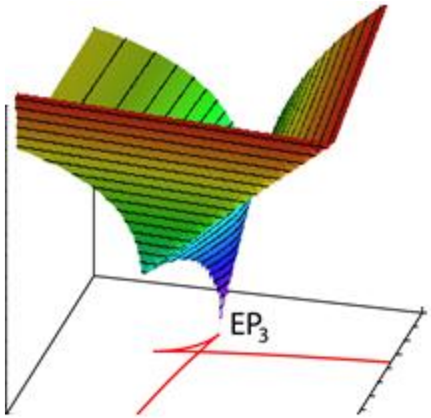
ZIEGLER-BOTTEMA DISSIPATION-INDUCED INSTABILITY AND RELATED TOPICS

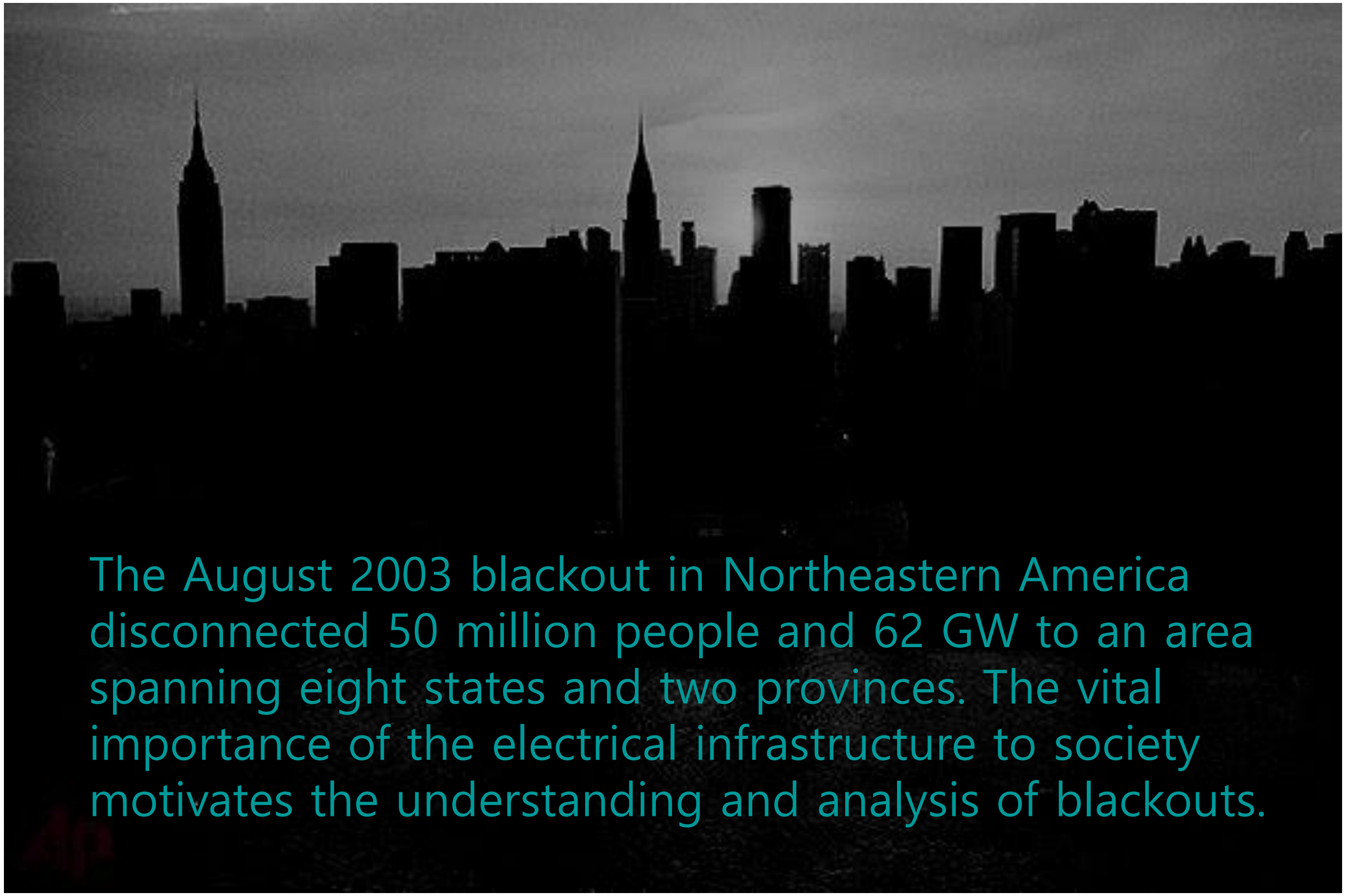


Oleg Kirillov

HELMHOLTZ ZENTRUM DRESDEN ROSSENDORF

Contents

Blackout in Northeastern America in 2003



The August 2003 blackout in Northeastern America disconnected 50 million people and 62 GW to an area spanning eight states and two provinces. The vital importance of the electrical infrastructure to society motivates the understanding and analysis of blackouts.

James BARRON Power Surge Blacks Out Northeast The New York Times, August 15, 2003

Blackouts and the inter-area oscillations

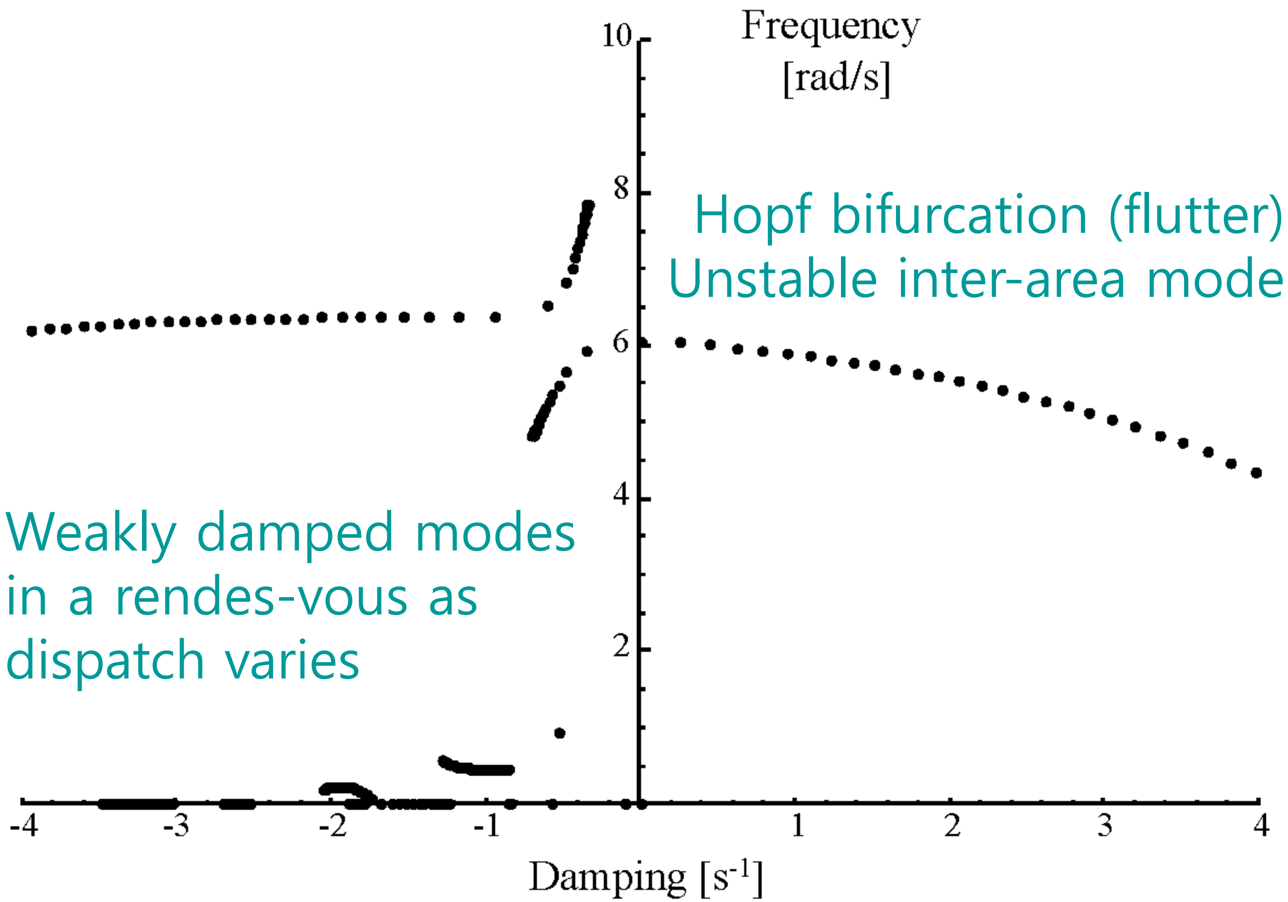
- Fact: Power transactions are increasing in volume and variety.
  Reason: Large amounts of money to be made in exploiting geographic differences in power prices.
  Limitation: The onset of electro-mechanical inter-area oscillations.

- Disturbance: Synchronous (AC) generators rotors swing back and forth in search for a new operating point; while swinging, generators in one geographical area form a coherent group which oscillates against another coherent group in another geographical area.
  The operation of the power system in the presence of a lightly damped inter-area mode is very difficult.

- Consequence: High power fluctuations on tie-lines that interconnect the areas. Even small variation of parameters may destabilize the oscillations and eventually lead to the power system collapse.
  The inter-area oscillations have been involved in many blackouts.

Andrzej ADAMCZYK Damping of low frequency power system oscillations with wind power plants. Department of Energy Technology, Aalborg University, 2012, 239 pp.

Modal resonance in a 3-bus power system



Ian DOBSON et al. Is strong modal resonance a precursor to power system oscillations? IEEE Trans. Circ. Syst. I 48 (2001) 340-349

Aleksandar ZECEVIC, Dejan MILJKOVIC The effects of generation redispatch on Hopf bifurcations in electric power systems. IEEE Trans. Circ. Syst. I: 49 (2002) 1180-1186

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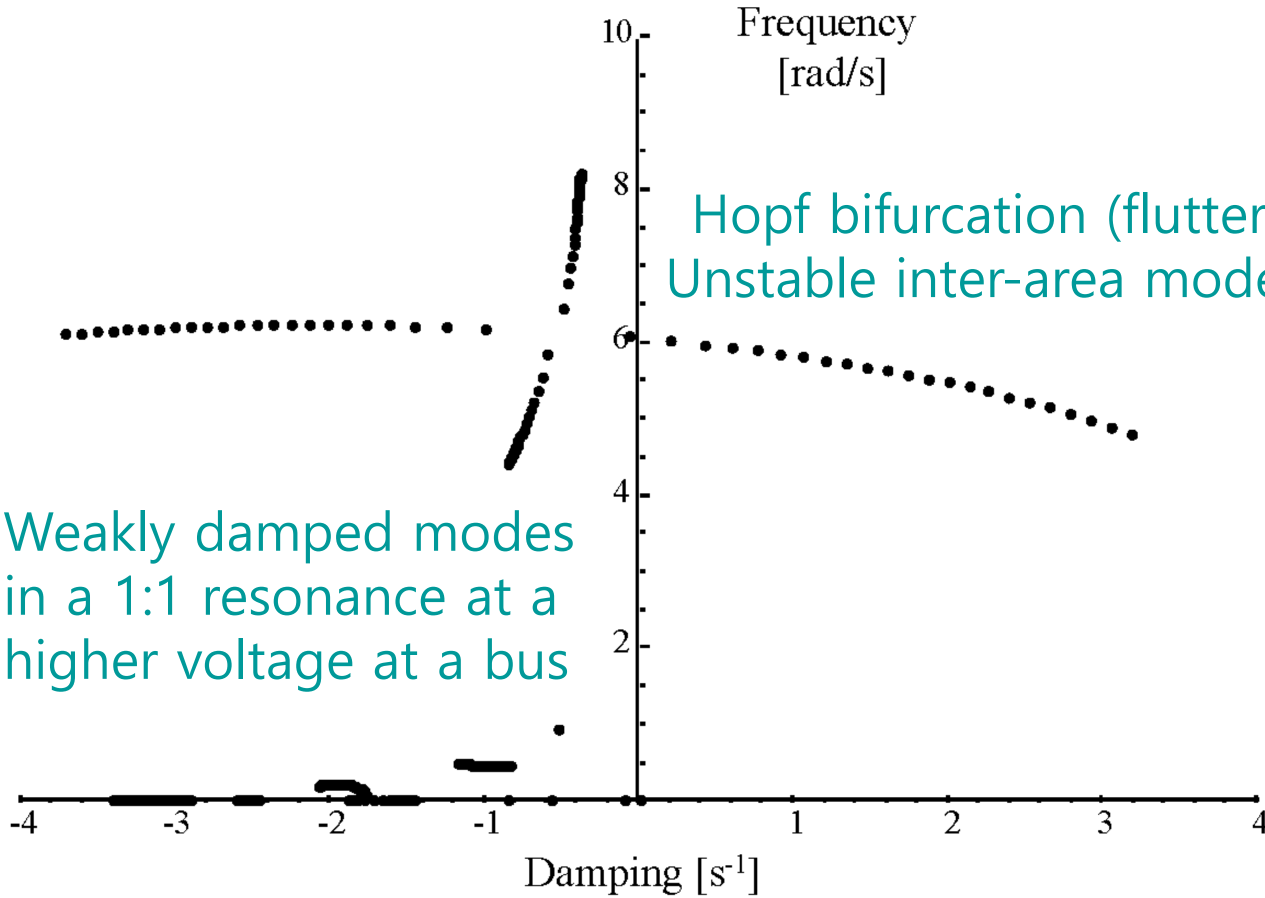

Hopf bifurcation (flutter)
Unstable inter-area mode

Weakly damped modes in a 1:1 resonance at a higher voltage at a bus

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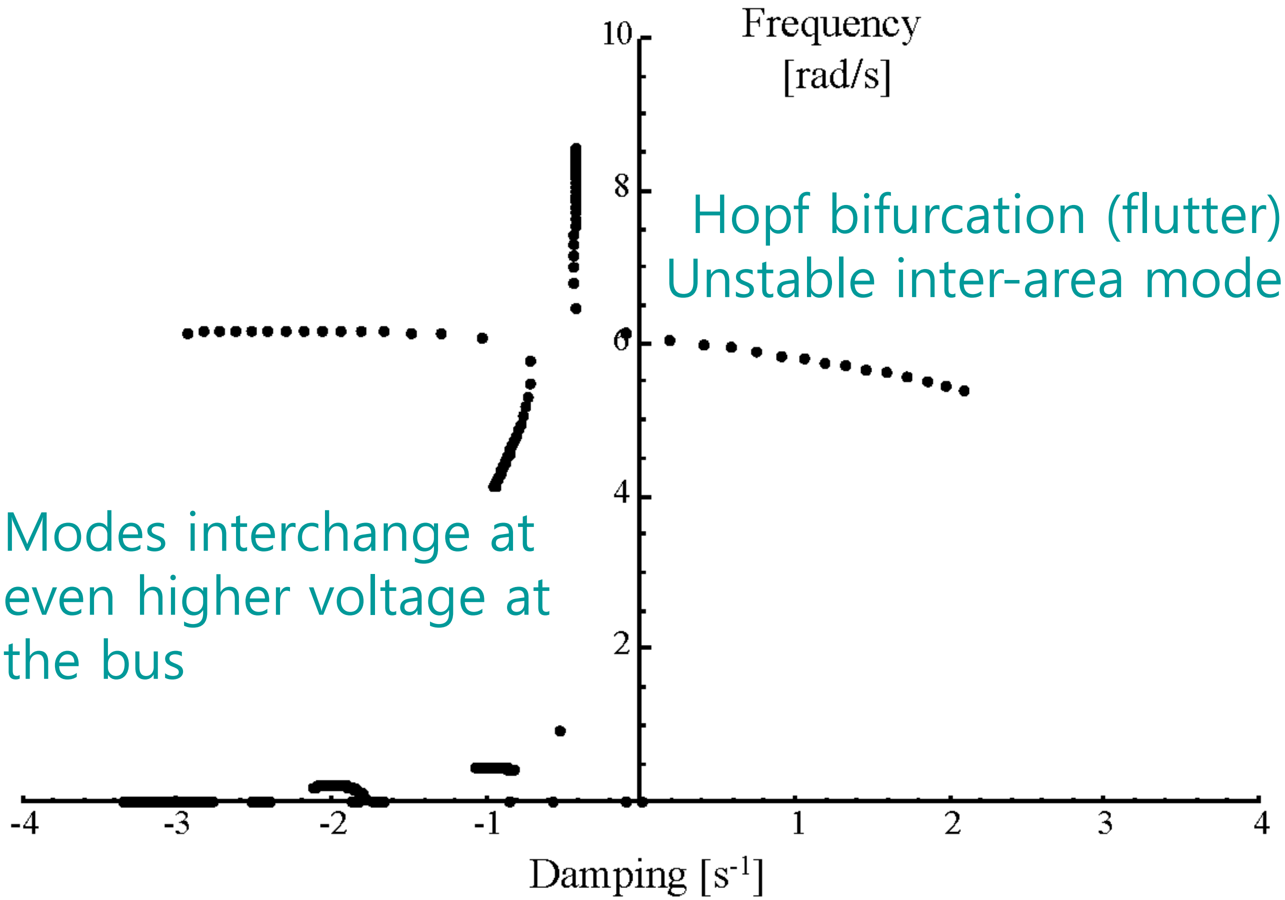

Hopf bifurcation (flutter)
Unstable inter-area mode

Modes interchange at even higher voltage at the bus

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Interlude

„Unfortunately, it is quite common for an eigenvalue which is moving steadily towards a positive growth rate to suffer a sudden change of direction and subsequently fail to become unstable.

Similarly, it happens that modes which initially become more stable as a parameter increases change direction and subsequently achieve instability.

It is believed that these changes of direction are due to the nearby presence of multiple-eigenvalue points."

Chris JONES Multiple eigenvalues and mode classification in plane Poiseulle flow.
Quart. J. Mech. Appl. Math. 41 (1988) 363-382

The classical power system model

$$M\ddot{\delta} + D\dot{\delta} + f_1(\delta, \phi, V, \mu) = 0, \quad f_2(\delta, \phi, V, \mu) = 0$$

M, symmetric matrix of generator rotor inertias;
D, symmetric damping matrix; δ, vector of generator internal bus angles; ϕ, vector of load bus angles; V, vector of constant power bus loads; μ, vector of network and load parameters

Non-symmetric matrix *K* of positional (circulatory) forces:

$$K^T \neq K := D_\delta f_1 - \begin{pmatrix} D_\phi \\ D_V \end{pmatrix} \begin{pmatrix} D_\phi \\ D_V \end{pmatrix}^{-1} D_\delta f_2$$

Linearized dynamics near the equlibrium $(\delta^*, \phi^*, V^*, \mu^*)$:

$$M\ddot{x} + D\dot{x} + Kx = 0, \quad x = \delta - \delta^*$$

This is a damped circulatory system

Harry KWATNY, Xiao-Ming YU Energy analysis of load-induced flutter instability in classical models of electric power networks. IEEE Trans. Circ. Syst. I 36 (1989) 1544-1557

Friction-induced whirl instability in an aircraft brake

Another example of a weakly damped circulatory system

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}_{\text{pressure}} + \mathbf{F}^{\text{NL}}(\mathbf{x}) \tag{20}$$

where $\ddot{\mathbf{x}}$, $\dot{\mathbf{x}}$ and $\mathbf{x}$ are the acceleration, velocity, and displacement response 15-dimensional vectors of the degrees-of-freedom, respectively. $\mathbf{M}$ is the mass matrix, $\mathbf{C}$ is

Fig. 2. Aircraft brake system.

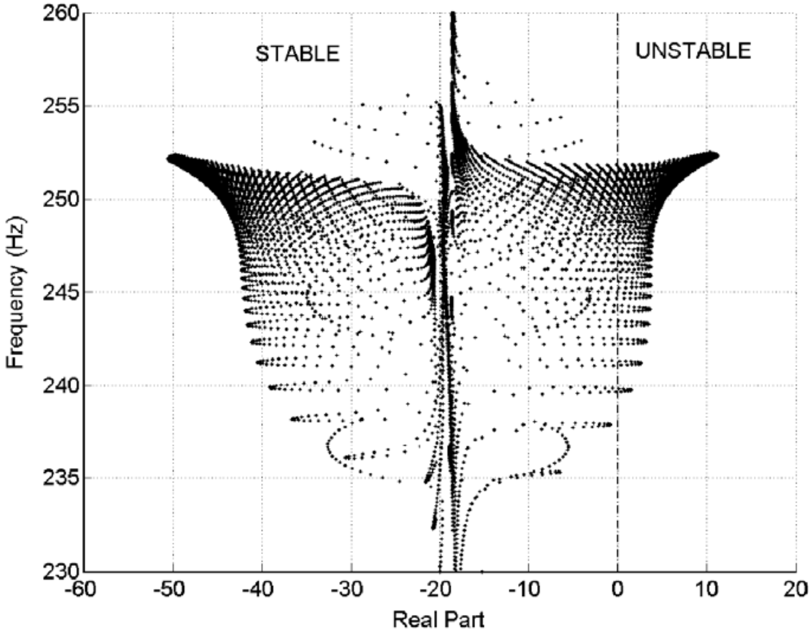


Fig. 11. Evolution of the imaginary and real part for the stable unstable mode versus the pressure and the friction coefficient in the complex plane.

J.-J. SINOU et al. Stability analysis and non-linear behaviour of structural systems using the complex non-linear modal analysis Computers and Structures 84 (2006) 1891–1905

Destabilizing effects of weak damping

1878 Kelvin

stability of rotating ellipsoidal shells containing fluid

1880 Greenhill

prolate shells unstable, oblate stable

1942 Sobolev

instability of chemical artillery shells

1944 Pontryagin, 1950 Krein

Hilbert space with indefinite metric, Krein collision, Krein signature

1960 Sturrock

dissipation-induced instability of negative energy modes in Hamiltonian systems

1883 Greenhill

buckling of a screw-shaft of a steamer

1927 Nicolai

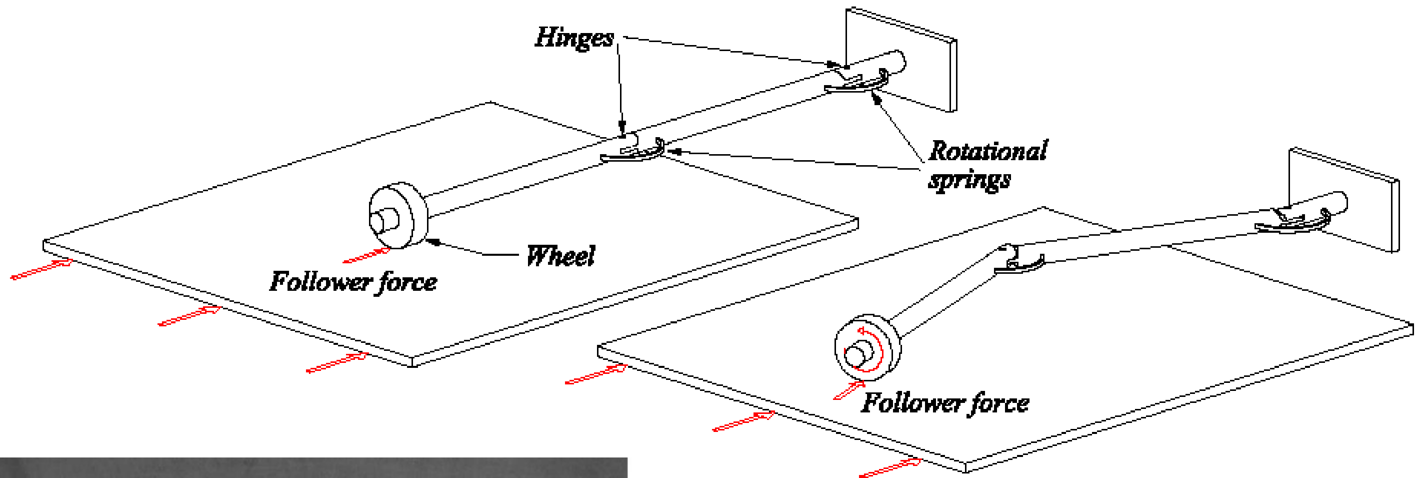
buckling and flutter of shafts under compression and (also follower) torque

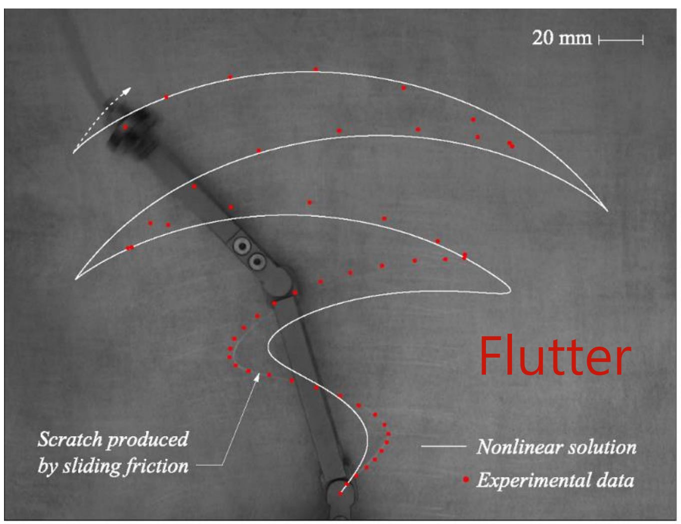
1952 Ziegler

flutter of rods under follower force, **destabilization paradox due to small dissipation in circulatory systems**

Oleg N. Kirillov, http://www.onkirillov.com

Ziegler's pendulum: Experimental realization





The non-conservative follower force originates through a wheel of negligible mass mounted at the top of the double pendulum and constrained to slide against a moving frictional plane

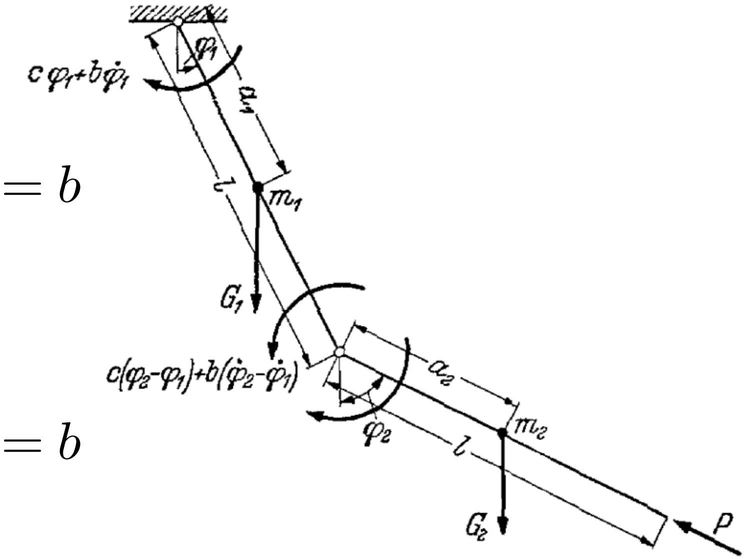
Davide BIGONI, Giovanni NOSELLI, Experimental evidence of flutter and divergence instabilities induced by dry friction, Journal of the Mechanics and Physics of Solids, 59 (2011) 2208–2226

Destabilization paradox in circulatory systems

- Ziegler's pendulum

$$b_1 = b$$

$$b_2 = b$$



Hans ZIEGLER Die Stabilitätskriterien der Elastomechanik Ing.-Arch. 20 (1952) 49-56

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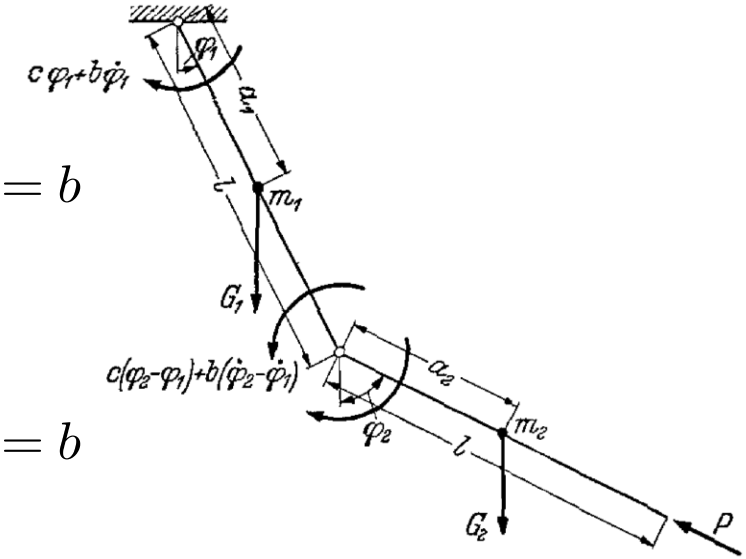

$$b_2 = b$$

- Follower force, P

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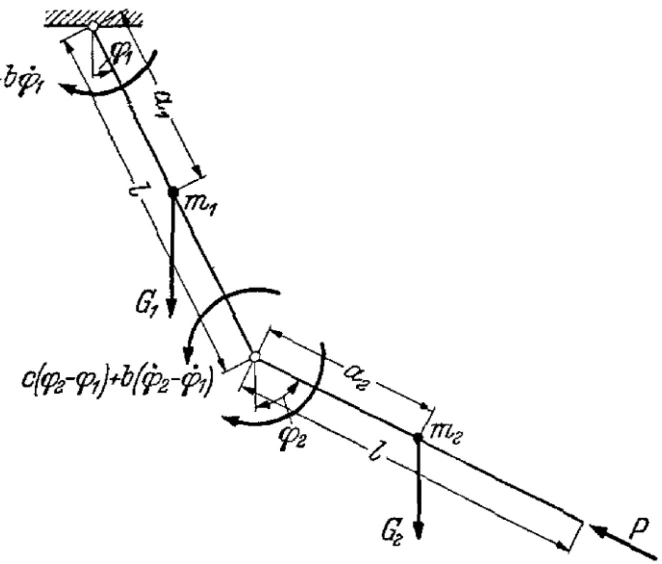

- Follower force, P
- Stiffness, c

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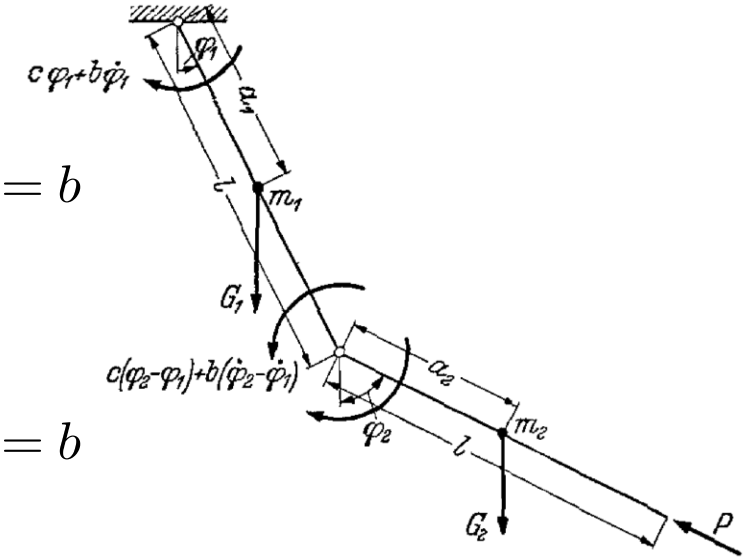

$b_2 = b$

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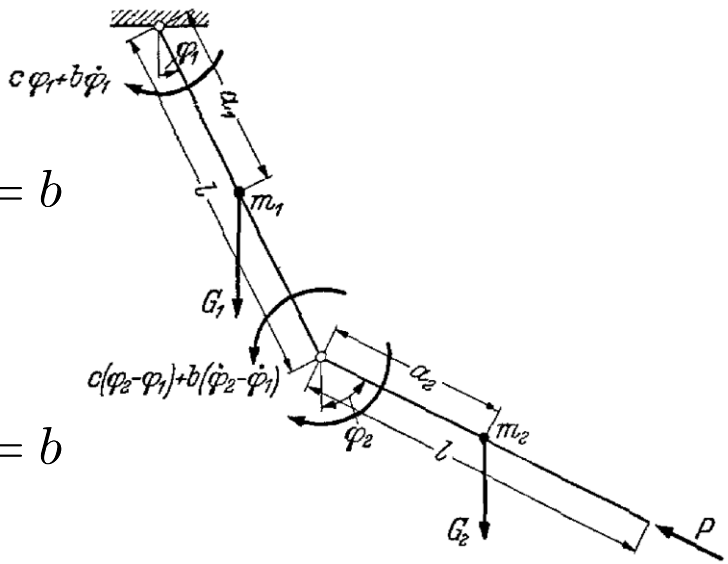

- Follower force, *P*
- Stiffness, *c*
- Damping, *b*

- Stability of the vertical equilibrium

b=0: Lyapunov stability for $P<P_k$

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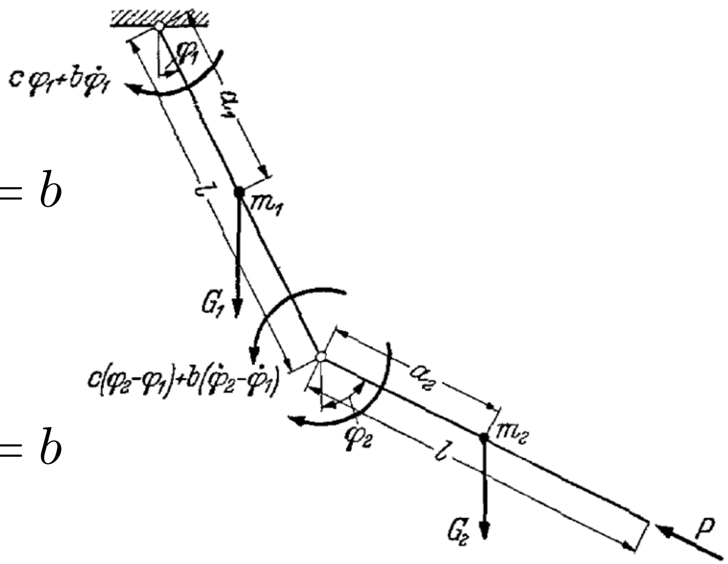

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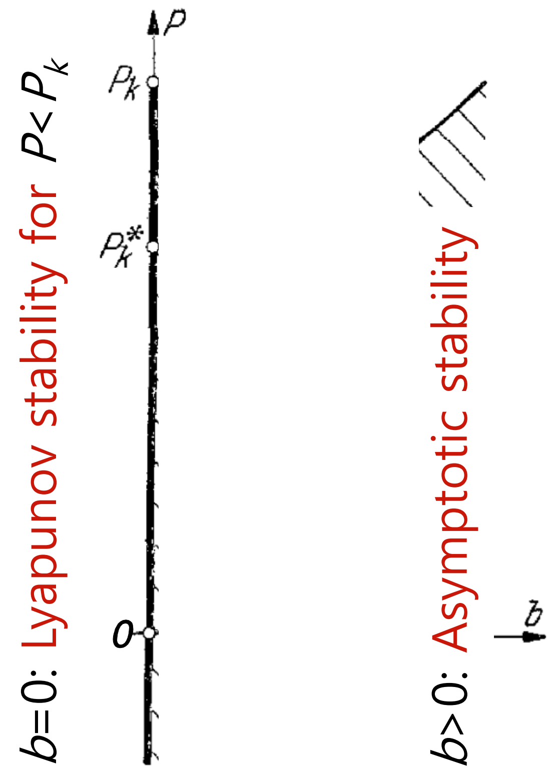

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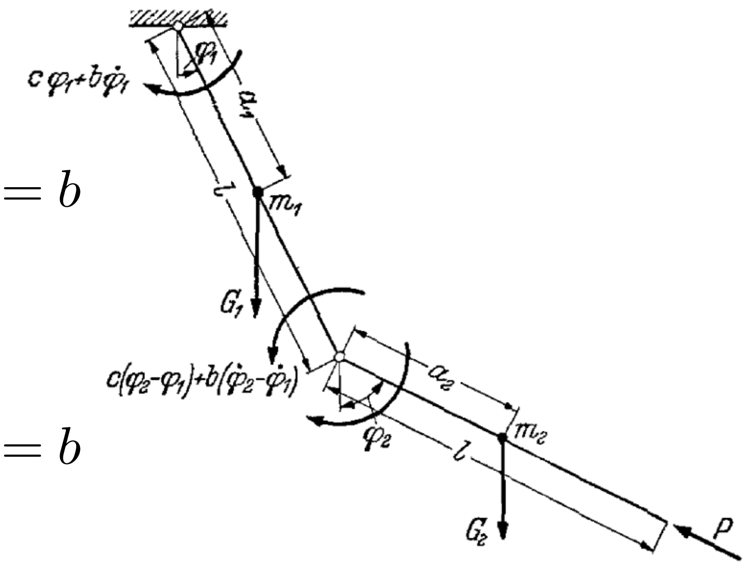

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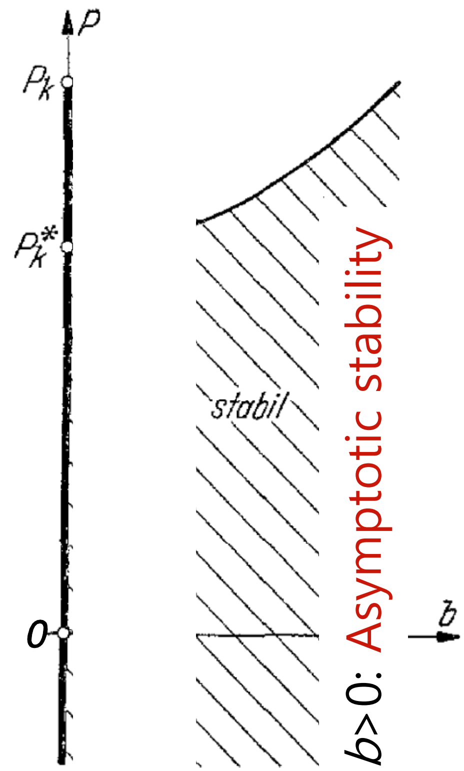

b>0: Asymptotic stability

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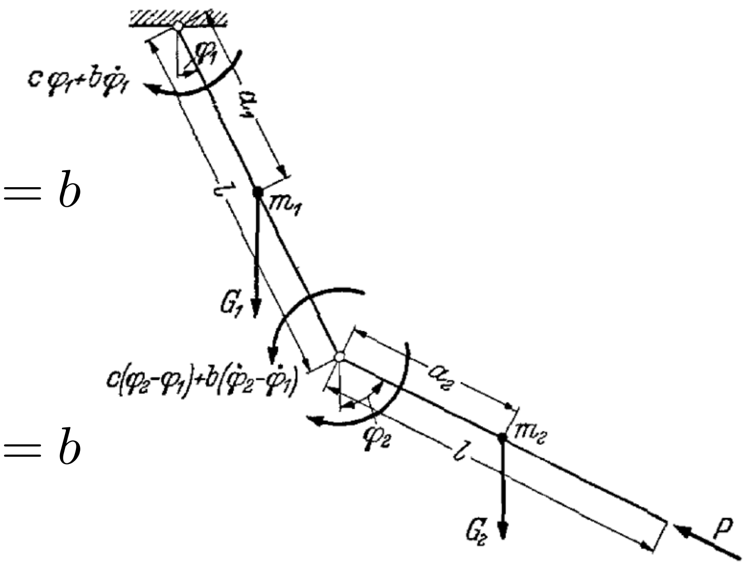

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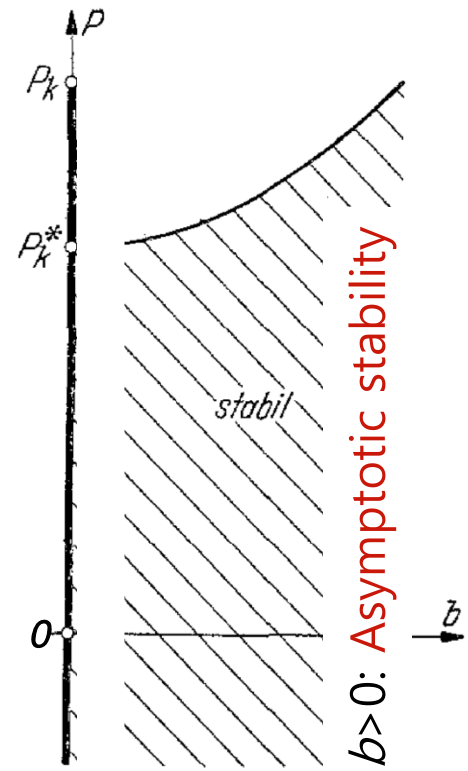

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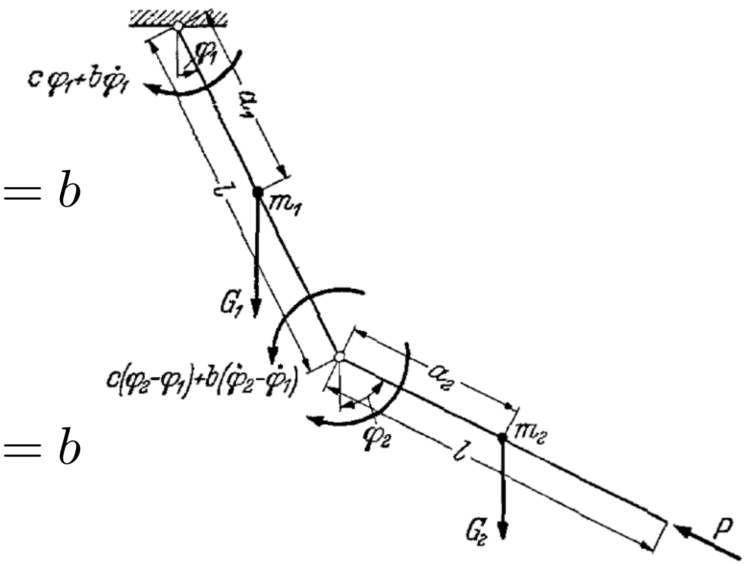

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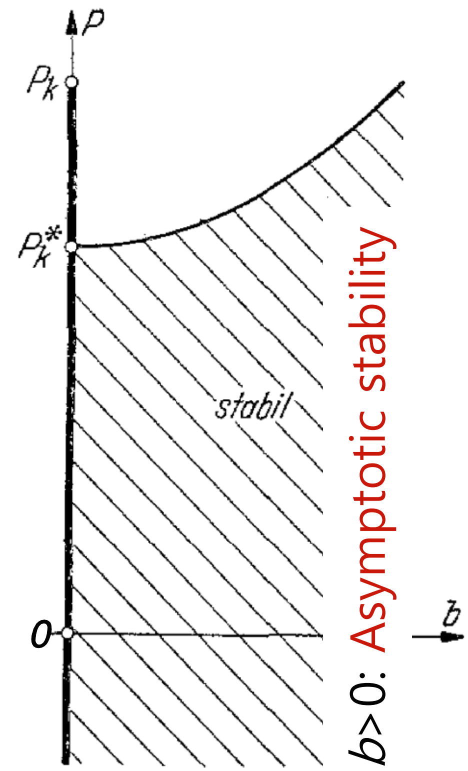

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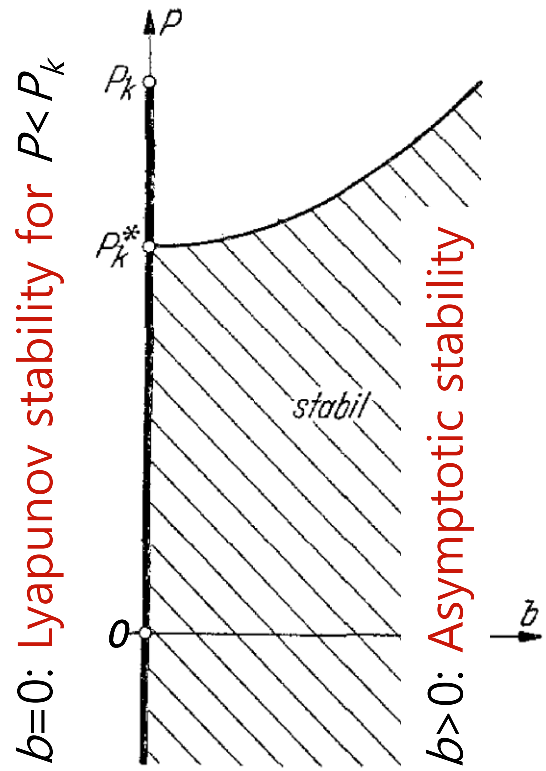
Destabilization paradox in circulatory systems

- Ziegler's pendulum

$$m_1 = 2m, \quad m_2 = m$$

$$a_1 = a_2 = l, \quad b_1 = b_2 = b$$

- Stability of the vertical equilibrium



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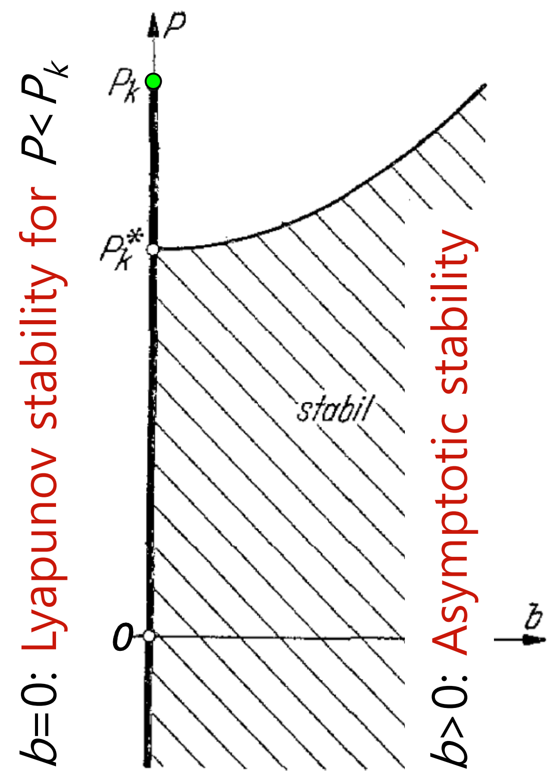

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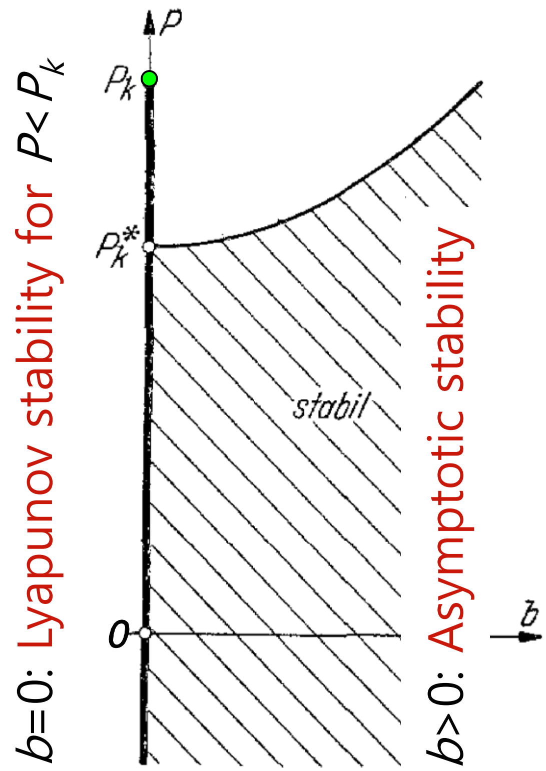

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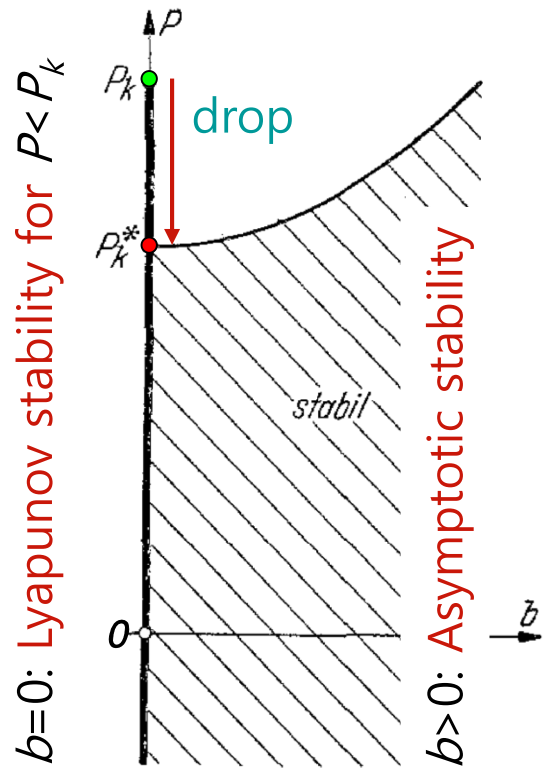
$$P_k^* = \lim_{b \to 0} P(b) = \frac{41}{28}\frac{c}{l} \approx 1.464\frac{c}{l} < P_k$$

- Stability of the vertical equilibrium



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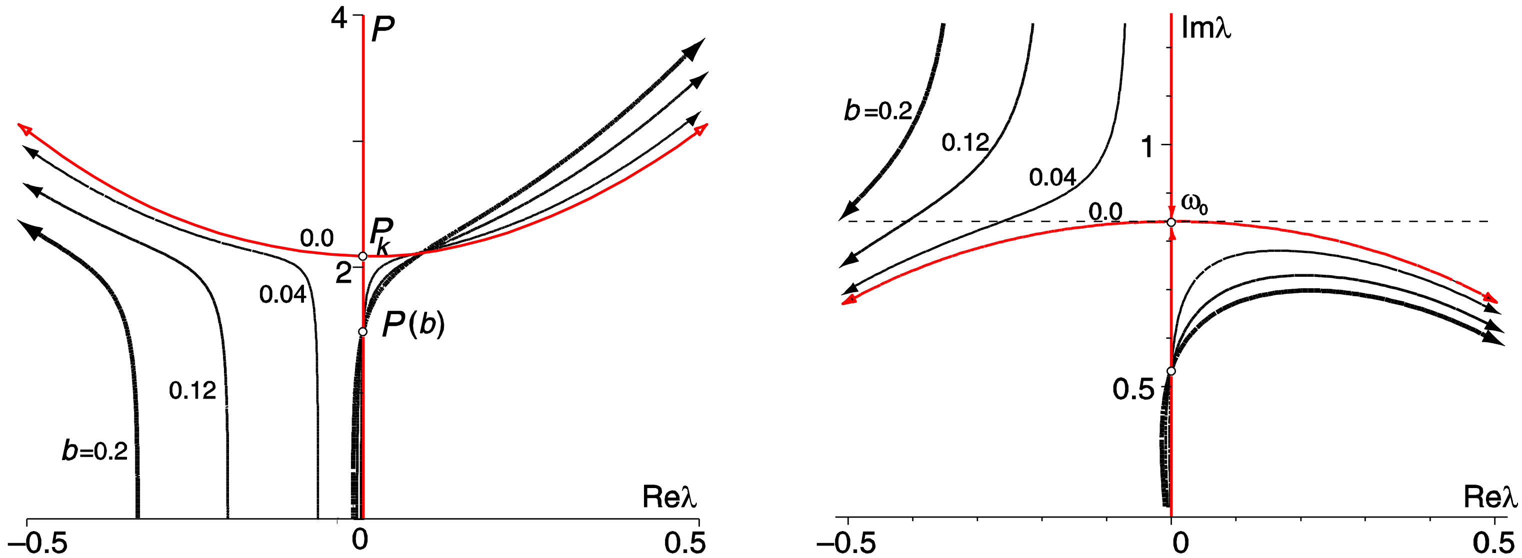
Destabilization paradox in circulatory systems

Imperfect modal resonance in the presence of damping



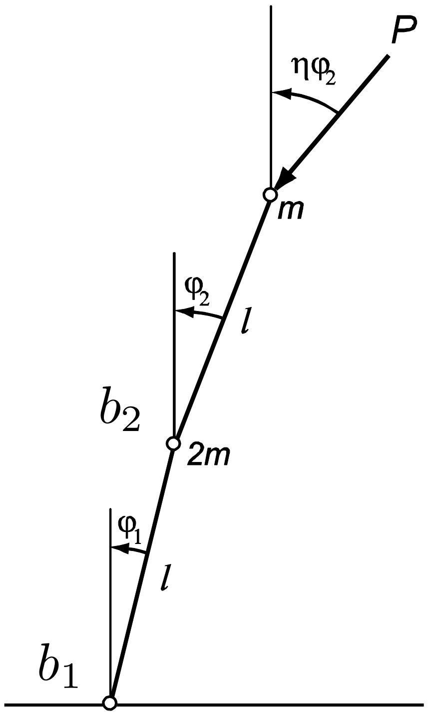
Imaginary eigenvalues get positive real increments under a dissipative perturbation

Hans ZIEGLER Die Stabilitätskriterien der Elastomechanik Ing.-Arch. 20 (1952) 49-56

1966 Herrmann & Jong

Ziegler's pendulum with the partially follower force

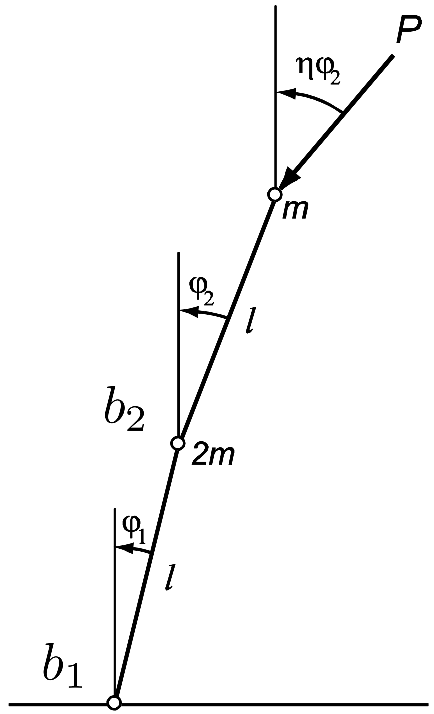
$$\begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix} \ddot{\mathbf{x}} + \begin{pmatrix} b_1 + b_2 & -b_2 \\ -b_2 & b_2 \end{pmatrix} \dot{\mathbf{x}} + \begin{pmatrix} 2-p & \eta p - 1 \\ -1 & 1-(1-\eta)p \end{pmatrix} \mathbf{x} = 0$$

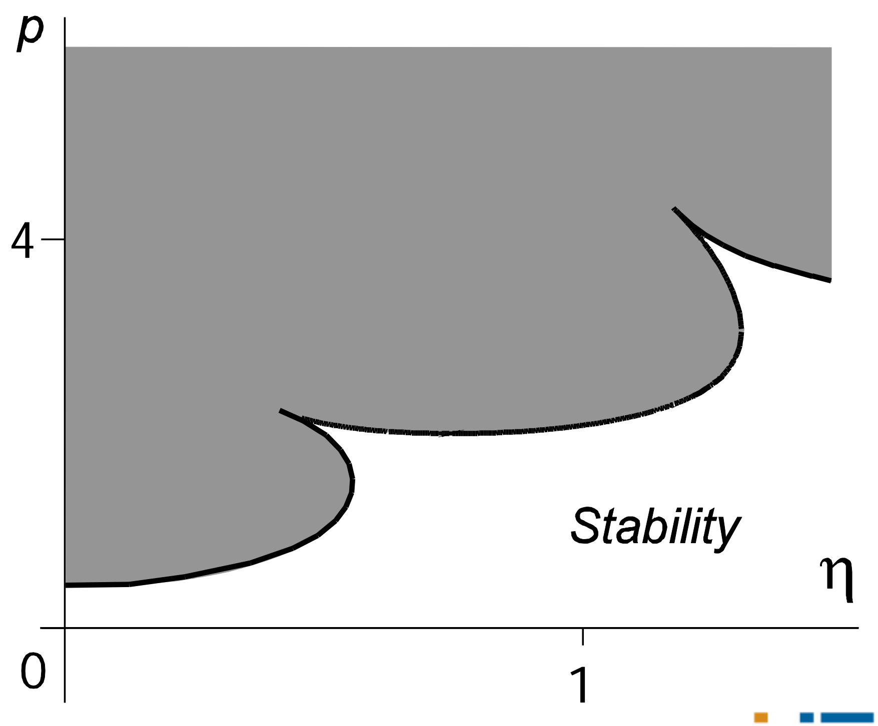


1966 Herrmann & Jong

Ziegler's pendulum with the partially follower force

Undamped instability domain ($b_1=0$, $b_2=0$)





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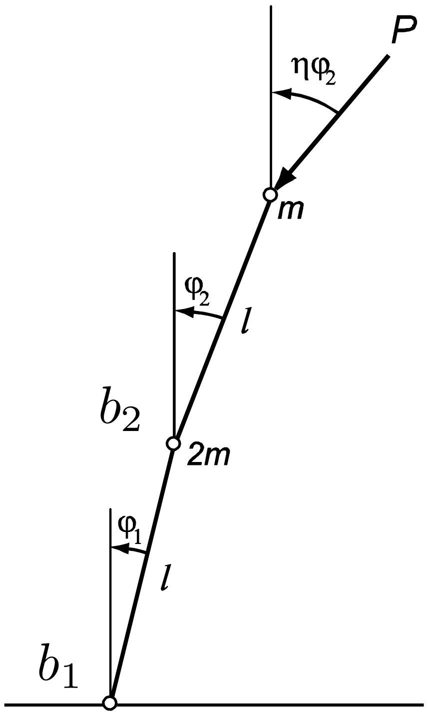
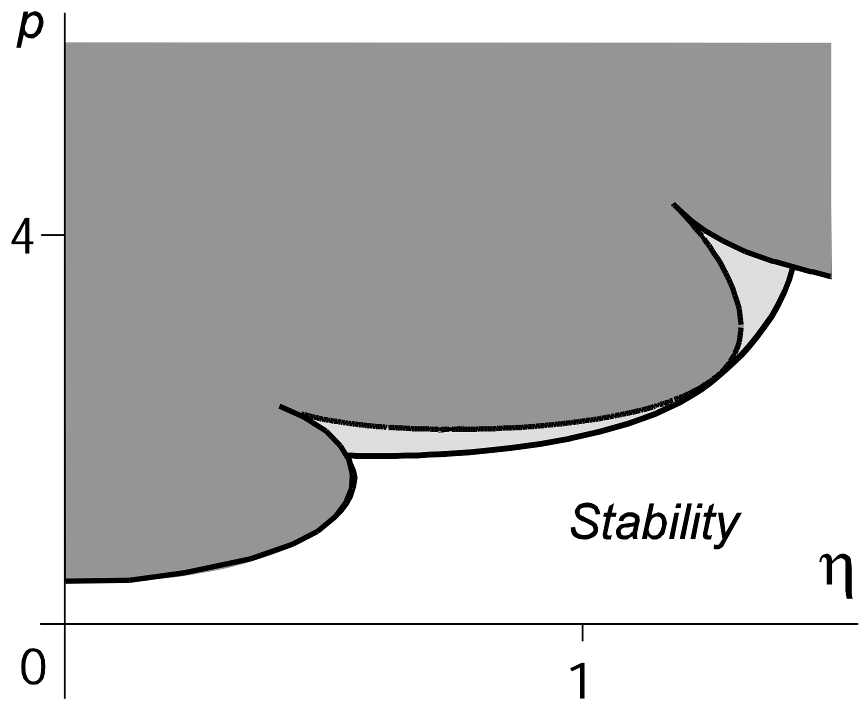
Ziegler's pendulum with the partially follower force

Undamped instability domain ($b_1=0$, $b_2=0$)
is not in the limit
of vanishing damping ($b_2=0.3b_1$, $b_1 \to 0$)



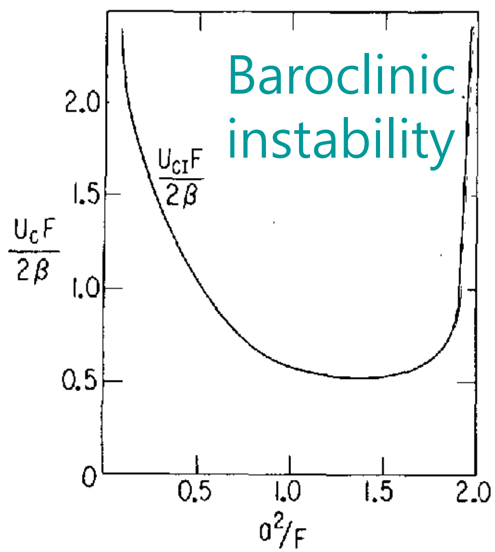


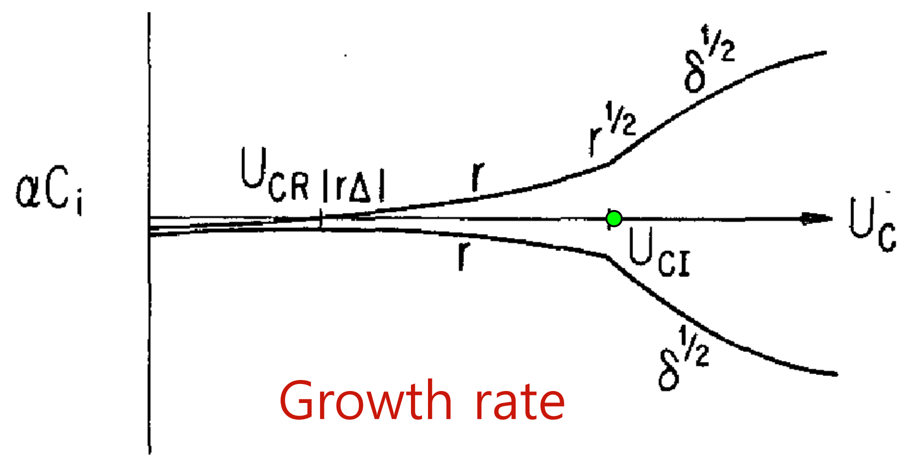
1961 Holopainen, 1977 Romea

Ekman layer dissipation enhances the baroclinic instability

Inviscid instability ($r = 0$)

$$U_{cI} = \frac{2\beta F}{a^2\sqrt{4F^2 - a^4}}$$





Oleg N. Kirillov, http://www.onkirillov.com

1961 Holopainen, 1977 Romea

Ekman layer dissipation enhances the baroclinic instability

Vanishing viscosity ($r \to 0$)

$$U_{cR} = \frac{2\beta F}{a(a^2+F)\sqrt{2F-a^2}}$$

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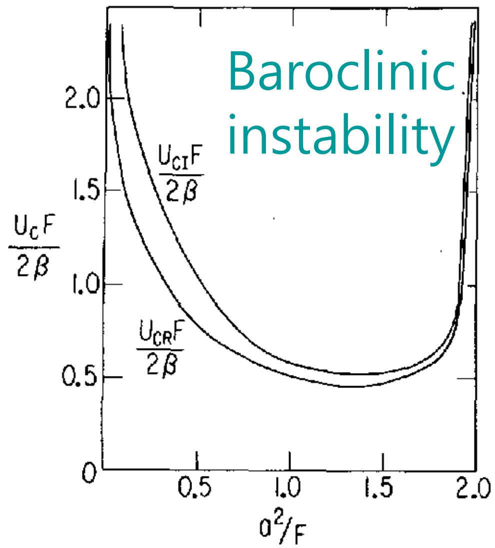

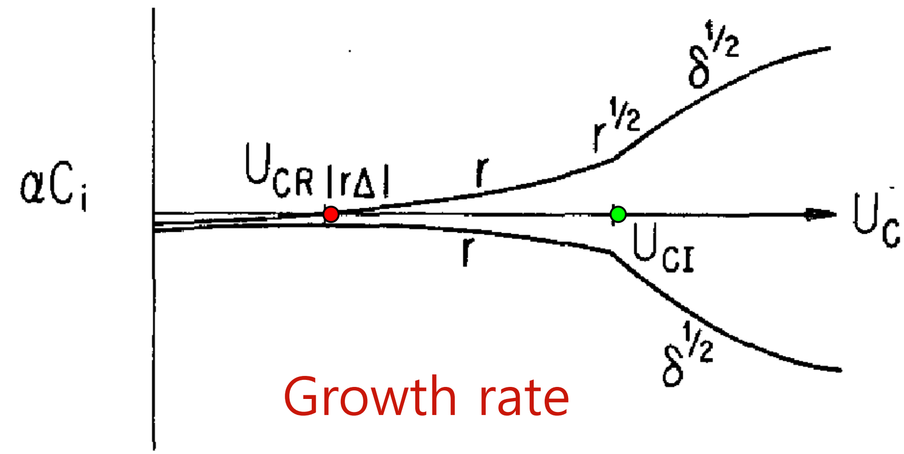

Dissipation-induced instabilities occur typically

- "Will the limit of stability corresponding to a gradually vanishing damping coincide in the limit with that found on the assumption that there is no damping?

V.V. BOLOTIN Nonconservative Problems of the Theory of Elastic Stability
Pergamon Press, Oxford, London, New York, Paris, 1963

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- These interesting aspects require further study for obtaining further, more definite, results."

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Pioneering works by Bottema (1956)

A linear non-conservative system with 2 d.o.f.

$$\ddot{\mathbf{x}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{x}} + (\mathbf{K}+\mathbf{N})\mathbf{x} = 0$$

Forces:

Dissipative, $\mathbf{D} = \begin{pmatrix} d_{11} & d_{12} \\ d_{12} & d_{22} \end{pmatrix}$ Potential, $\mathbf{K} = \begin{pmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{pmatrix}$

Gyroscopic, $\mathbf{G} = \begin{pmatrix} 0 & \Omega \\ -\Omega & 0 \end{pmatrix}$ Circulatory, $\mathbf{N} = \begin{pmatrix} 0 & \nu \\ -\nu & 0 \end{pmatrix}$

Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

Pioneering works by Bottema (1956)

A linear non-conservative system with 2 d.o.f.

$$\ddot{\mathbf{x}} + (\mathbf{D}+\mathbf{G})\dot{\mathbf{x}} + (\mathbf{K}+\mathbf{N})\mathbf{x} = 0$$

Characteristic polynomial:

$$\mathbf{x} = e^{\mu t}\mathbf{u}, \quad q(\mu) = \mu^4 + q_1\mu^3 + q_2\mu^2 + q_3\mu + q_4$$

$$q_1 = \mathrm{tr}\mathbf{D}, \quad q_3 = \mathrm{tr}\mathbf{K}\mathrm{tr}\mathbf{D} - \mathrm{tr}\mathbf{K}\mathbf{D} + 2\Omega\nu$$

$$q_2 = \mathrm{tr}\mathbf{K} + \det\mathbf{D} + \Omega^2, \quad q_4 = \det\mathbf{K} + \nu^2$$

Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

Pioneering works by Bottema (1956)

$$q(\mu) = \mu^4 + q_1\mu^3 + q_2\mu^2 + q_3\mu + q_4$$

Hurwitz condition:

$$q_i > 0, \quad q_2 > \frac{q_1^2 q_4 + q_3^2}{q_1 q_3}$$

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Pioneering works by Bottema (1956)

$$q(\mu) = \mu^4 + q_1\mu^3 + q_2\mu^2 + q_3\mu + q_4$$

$$\mu = c\lambda, \quad c = \sqrt[4]{q_4}, \quad a_i = \frac{q_i}{c^i}$$

Hurwitz condition:

$$q_i > 0, \quad q_2 > \frac{q_1^2 q_4 + q_3^2}{q_1 q_3}$$

Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

Pioneering works by Bottema (1956)

$$p(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1$$

$$\mu = c\lambda, \quad c = \sqrt[4]{q_4}, \quad a_i = \frac{q_i}{c^i}$$

Hurwitz condition:

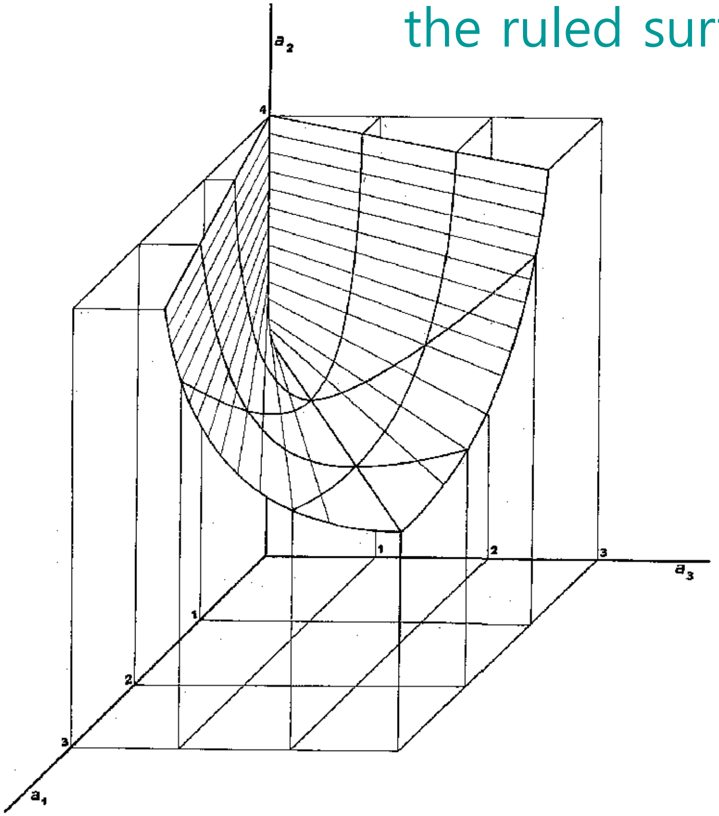
$$a_i > 0, \quad a_2 > 2 + \frac{(a_1 - a_3)^2}{a_1 a_3}$$

Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

Pioneering works by Bottema (1956)

$$p(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1$$

Asymptotic stability inside
the ruled surface



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Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

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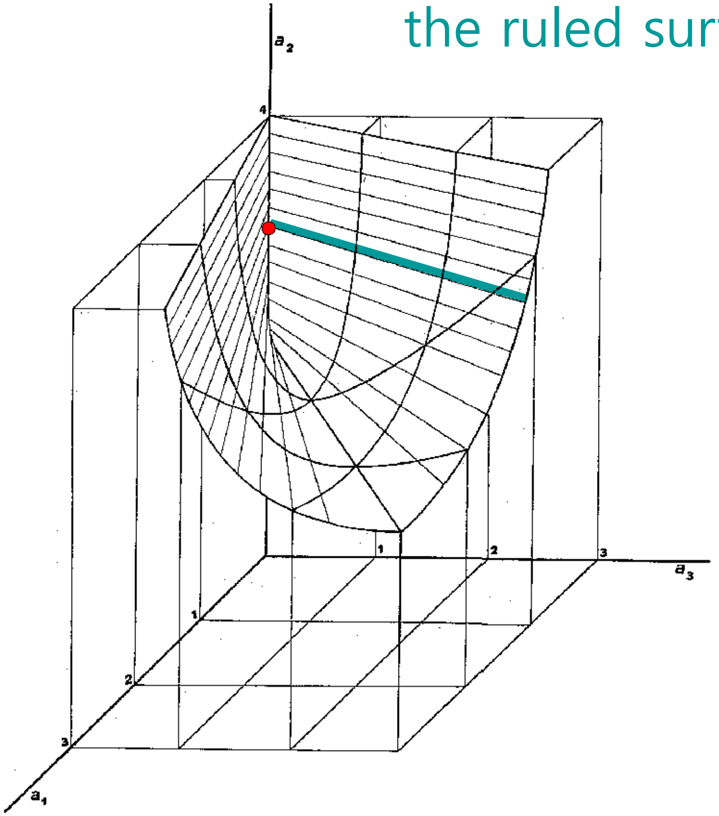

Hurwitz condition:

$$a_i > 0, \quad a_2 > 2 + \frac{(a_1 - a_3)^2}{a_1 a_3}$$

Generators:

$$a_3 = r a_1, \quad a_2 = r + \frac{1}{r}$$

Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

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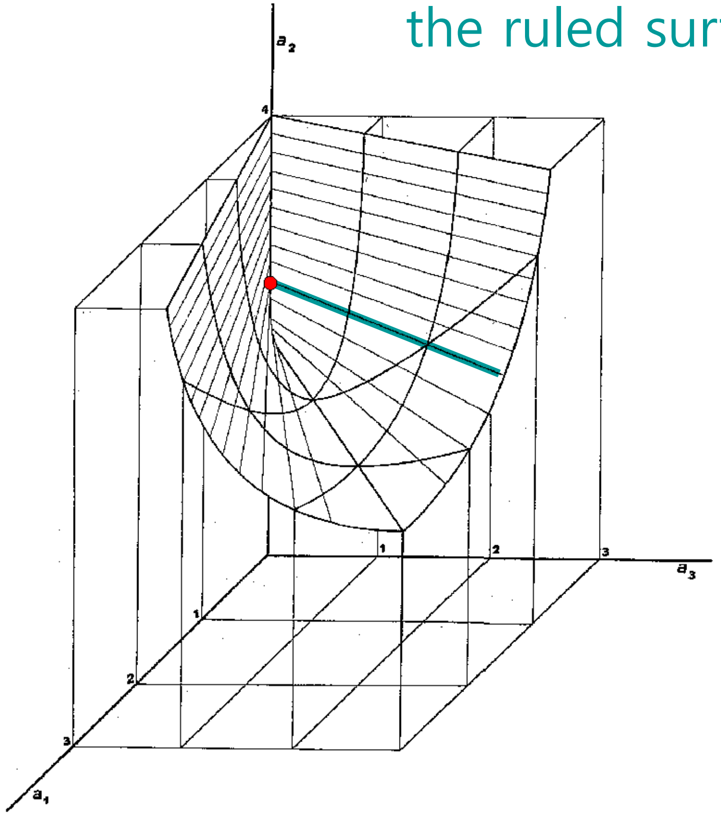
$$p(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1$$

Asymptotic stability inside the ruled surface



Hurwitz condition:

$$a_i > 0, \quad a_2 > 2 + \frac{(a_1 - a_3)^2}{a_1 a_3}$$

Generators:

$$a_3 = r a_1, \quad a_2 = r + \frac{1}{r}$$

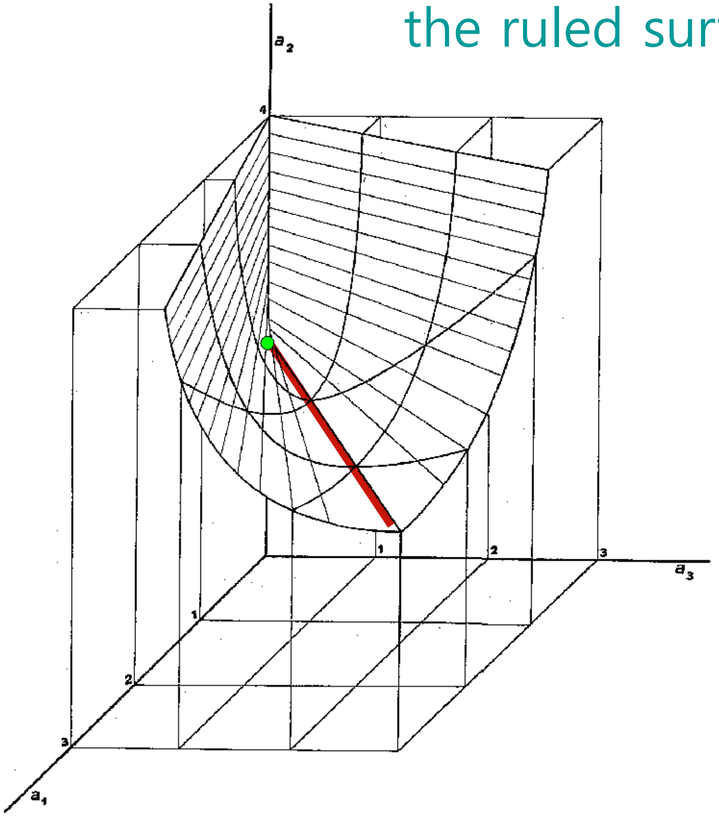
$$r \in (0, \infty)$$

Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

Pioneering works by Bottema (1956)

$$p(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1$$

Asymptotic stability inside the ruled surface



Hurwitz condition:

$$a_i > 0, \quad a_2 > 2 + \frac{(a_1 - a_3)^2}{a_1 a_3}$$

Generators:

$$a_3 = r a_1, \quad a_2 = r + \frac{1}{r}$$

$$r \in (0, \infty)$$

Minimum: $a_2 = 2$

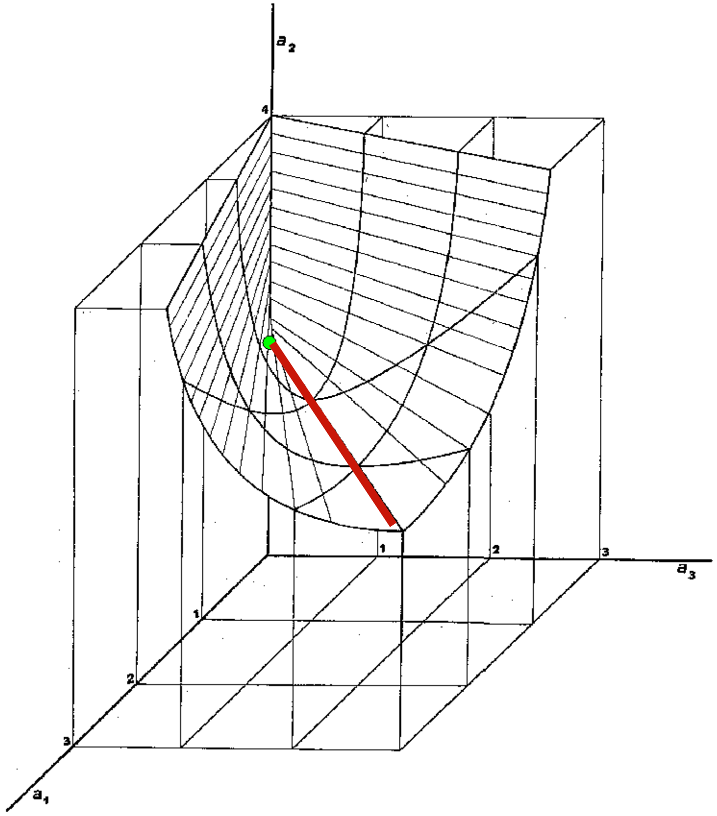
$$r = 1, \quad i.e. \quad a_3 = a_1$$

Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

Pioneering works by Bottema (1956)

$$a_2 = 2 + \frac{(a_1 - a_3)^2}{a_1 a_3} > 2, \quad a_3 \neq a_1$$

$$a_2 = 2, \quad a_3 = a_1$$



„Here is the discontinuity we mentioned above. It plays a part in questions regarding the stability of equilibrium.

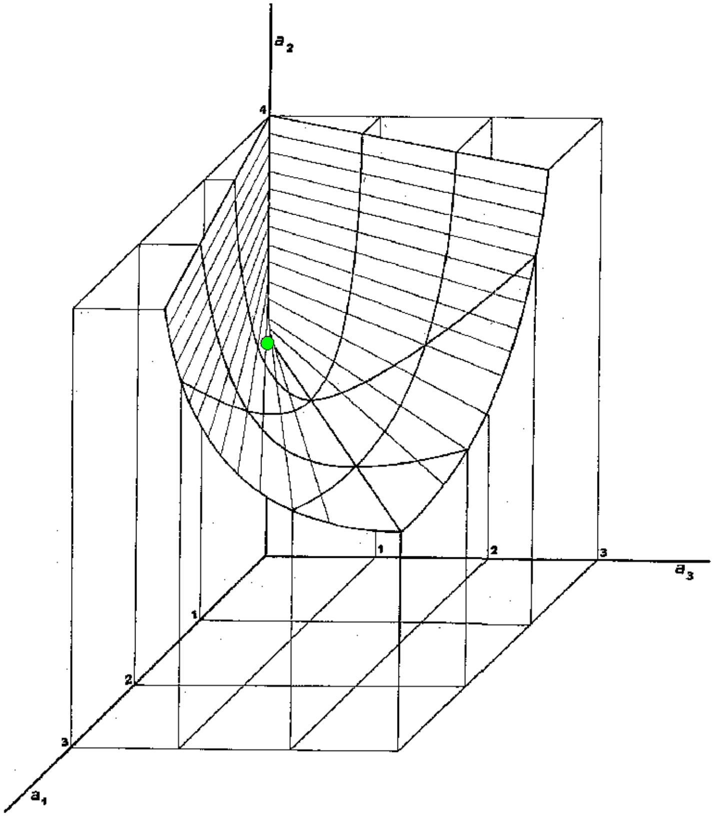
The coefficients a_1 and a_3 depend on the linear damping forces and it is well known that the stability condition may change in a discontinuous way if a very small damping vanishes at all.

The phenomenon may be illustrated by a geometrical diagram.“ Bottema, 1956

Oene BOTTEMA The Routh-Hurwitz condition for the biquadratic equation Indagationes Mathematicae, 18 (1956) 403-406

Generic singularities by Arnold (1971)

Double imaginary
root at the singular point:
λ=i



two-parameter families.[1] *These singularities can be listed to within diffeomorphism as follows:*

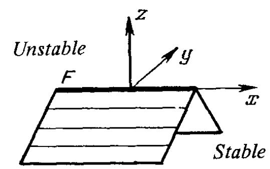


Fig. 4.14.

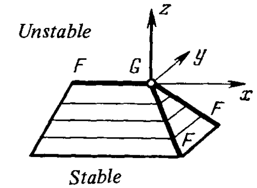


Fig. 4.15.

Two faces meeting along a ridge (F_i; Fig. 4.14): $z + |y| = 0$.

Three faces meeting at a corner ($G_{3,4,5}$; Fig. 4.15): $z + \max(x, |y|) = 0$.

Cuspidal point on a ridge (G_2; Fig. 4.17): $z + |\mathrm{Re}\sqrt{(x+iy)}| = 0$. (*This surface in* $\mathbf{R}^3$ *is diffeomorphic to that given by the equations* $XY^2 = Z^2$, *where* $Y \geqslant 0$.)

Node on a ridge (G_1; Fig. 4.16): $z + \lambda(x, y) = 0$, *where* λ *is the greatest real part of the roots of the equation* $\lambda^3 = x\lambda + y$. (*This surface in* $\mathbf{R}^3$ *is diffeomorphic to that given by* $X^2Y^2 = Z^2$, $X \geqslant 0$, $Y \geqslant 0$.)

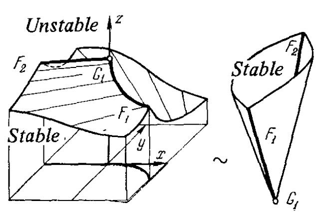


Fig. 4.16.

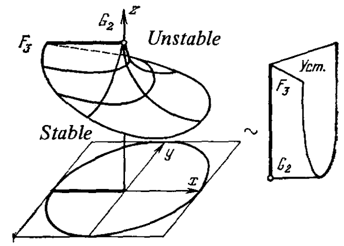


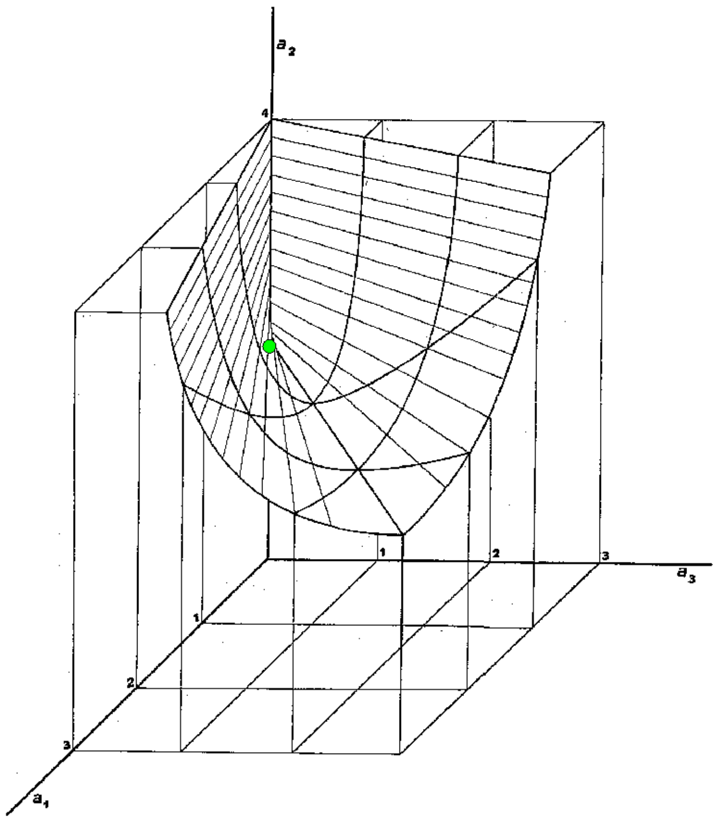
Fig. 4.17.

The acute angles of the stability boundary always point into the domain of instability.

V. I. ARNOLD On Matrices Depending on Parameters Russ. Math. Surv. 26 (1971) 29-43

Whitney umbrella singularity at $a_2=2,\ a_1=a_3=0$

Double imaginary root at the singular point: $\lambda=i$



$$\lambda^4+a_1\lambda^3+a_2\lambda^2+a_3\lambda+1$$

Companion matrix:

$$\mathbf{A}=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & -a_3 & -a_2 & -a_1 \end{pmatrix}$$

Jordan form:

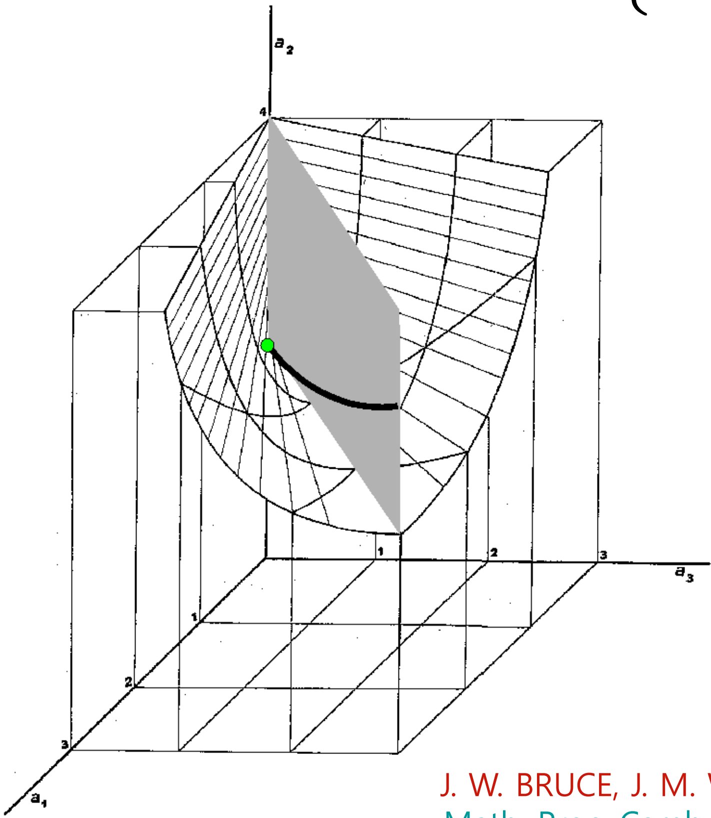
$$\mathbf{J}_2=\begin{pmatrix} i & 1 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & -i & 1 \\ 0 & 0 & 0 & -i \end{pmatrix}$$

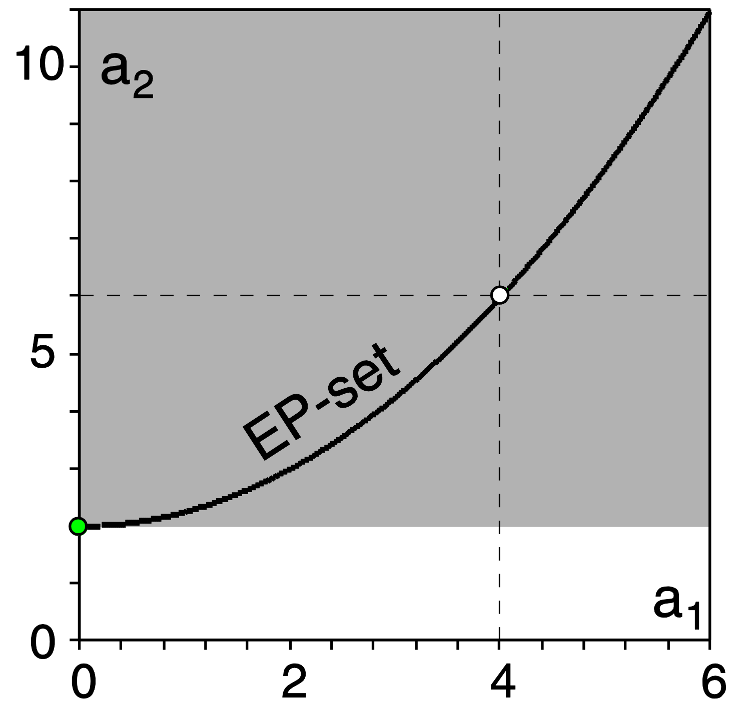
V. I. ARNOLD On Matrices Depending on Parameters Russ. Math. Surv. 26 (1971) 29-43

Tangent cone to the stability domain

Tangent cone: $\{(a_1, a_3, a_2): \quad a_1 = a_3, \quad a_1 > 0, \quad a_2 > 2\}$

EP-set: $\left\{(a_1, a_3, a_2): \quad a_1 = a_3, \quad a_2 = 2 + \frac{a_1^2}{4}\right\}$





J. W. BRUCE, J. M. WEST Functions on a crosscap.
Math. Proc. Camb. Philos. Soc. 123 (1998) 19–39.

Tangent cone to the stability domain

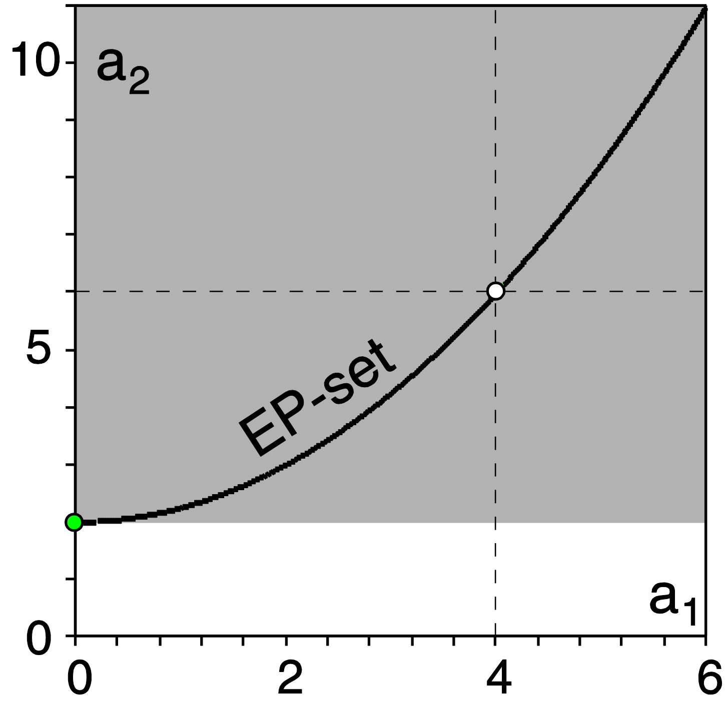
Tangent cone: $\{(a_1, a_3, a_2): \quad a_1 = a_3, \quad a_1 > 0, \quad a_2 > 2\}$

EP-set: $\left\{(a_1, a_3, a_2): \quad a_1 = a_3, \quad a_2 = 2 + \frac{a_1^2}{4}\right\}$

Multiple eigenvalues at the EP-set:

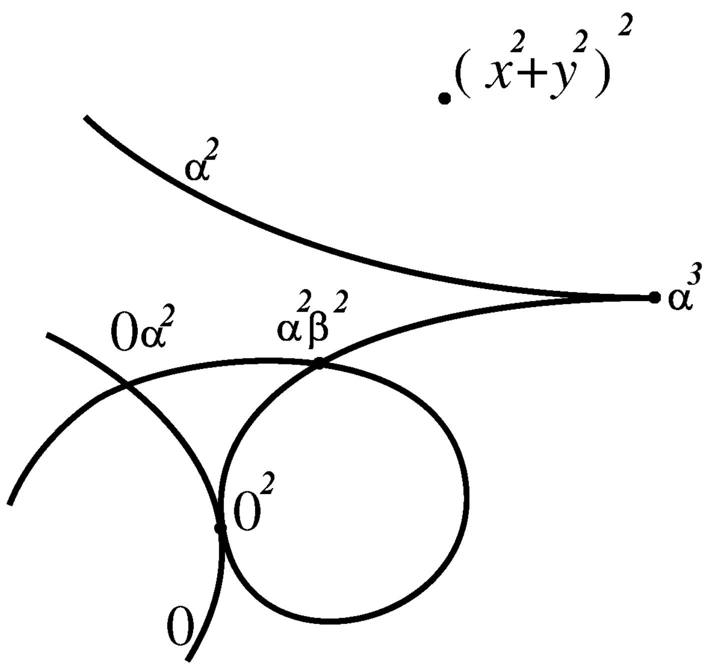
$$\lambda_1 = \lambda_2 = -\frac{a_1}{4} - \frac{1}{4}\sqrt{a_1^2 - 16}$$

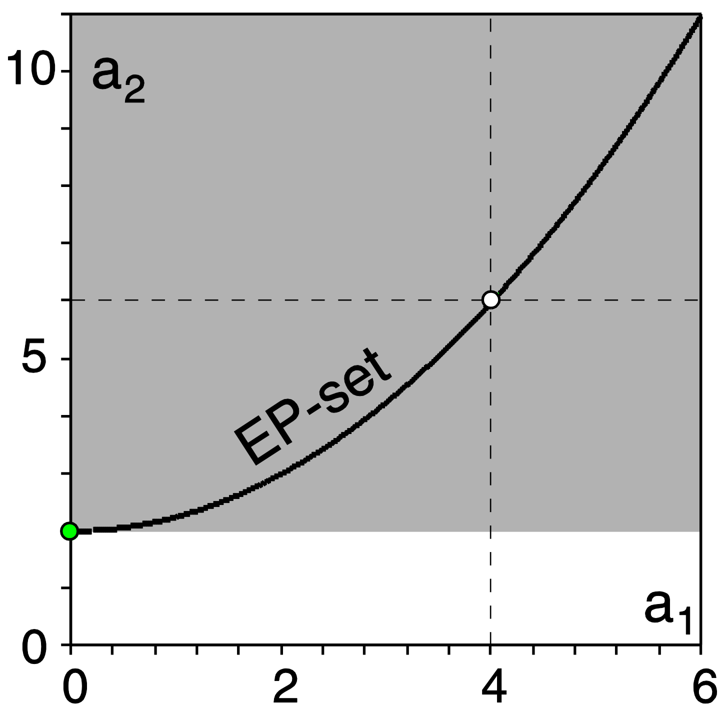
$$\lambda_3 = \lambda_4 = -\frac{a_1}{4} + \frac{1}{4}\sqrt{a_1^2 - 16}$$



Bifurcation diagrams of families of real matrices

Double eigenvalues x+iy with the Jordan block:
codim=2





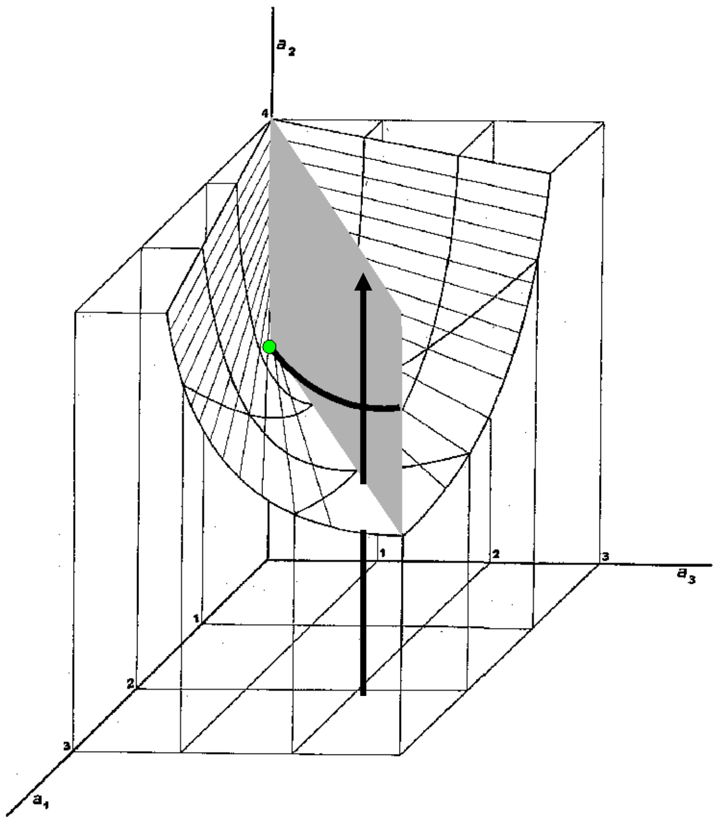
D.M. GALIN Real matrices depending on parameters. Uspekhi Mat. Nauk 27 (1972) 241–241.

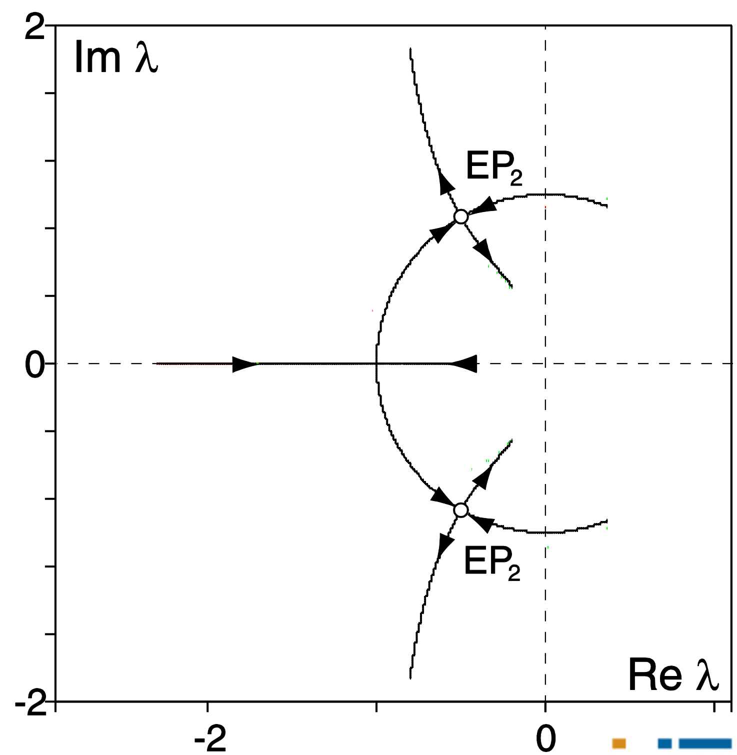
Movement of eigenvalues

Parameters change within the tangent cone

$$a_1 = a_3 = 2, \quad 0 \le a_2 \le 6$$

Collisions on the unit circle at exceptional points (EP_2), $\mathrm{Re}\lambda < 0$



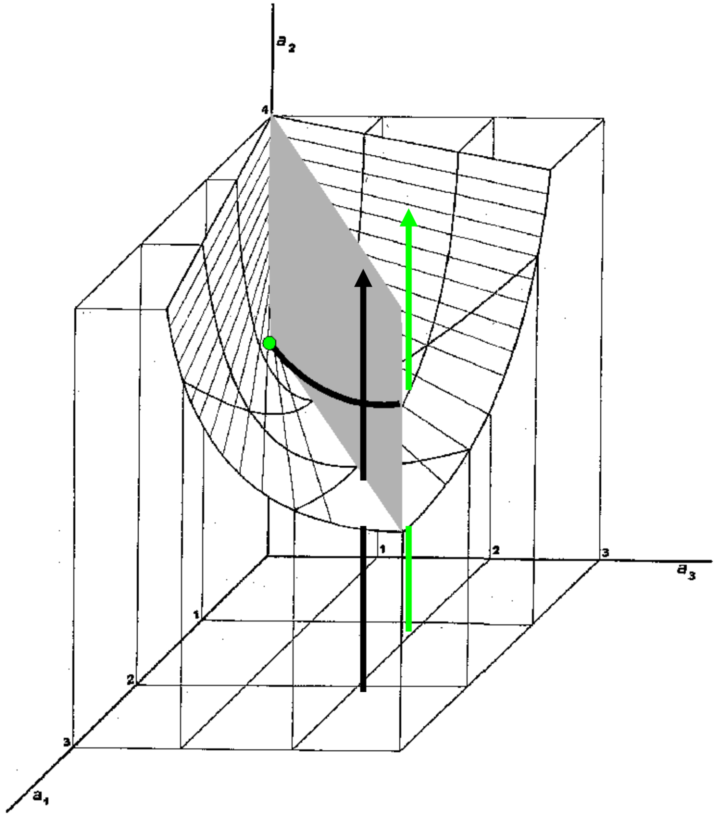


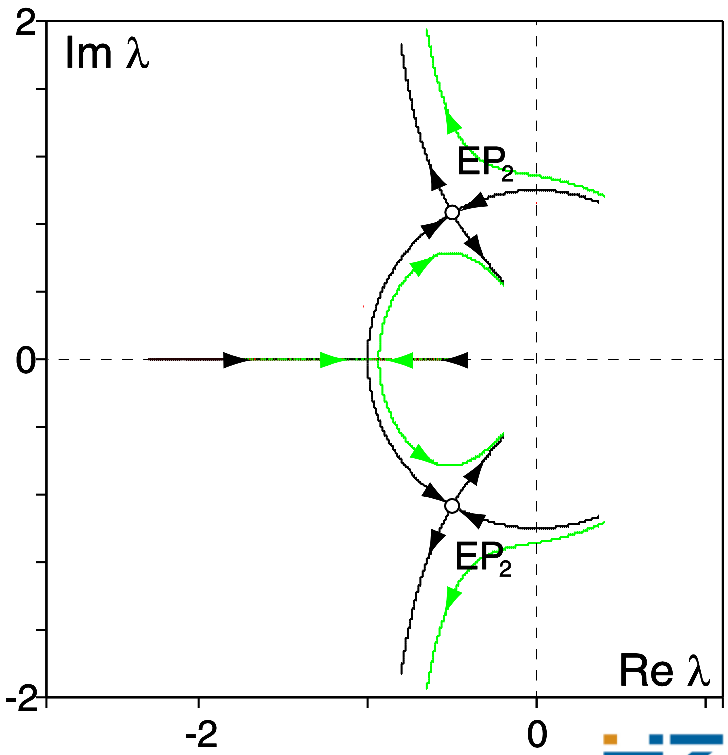
Imperfect merging of modes

Parameters change right to the tangent cone

$$a_1 = 1.7, \quad a_3 = 2, \quad 0 \le a_2 \le 6$$

Avoided crossings near EP_2, $\mathrm{Re}\lambda < 0$



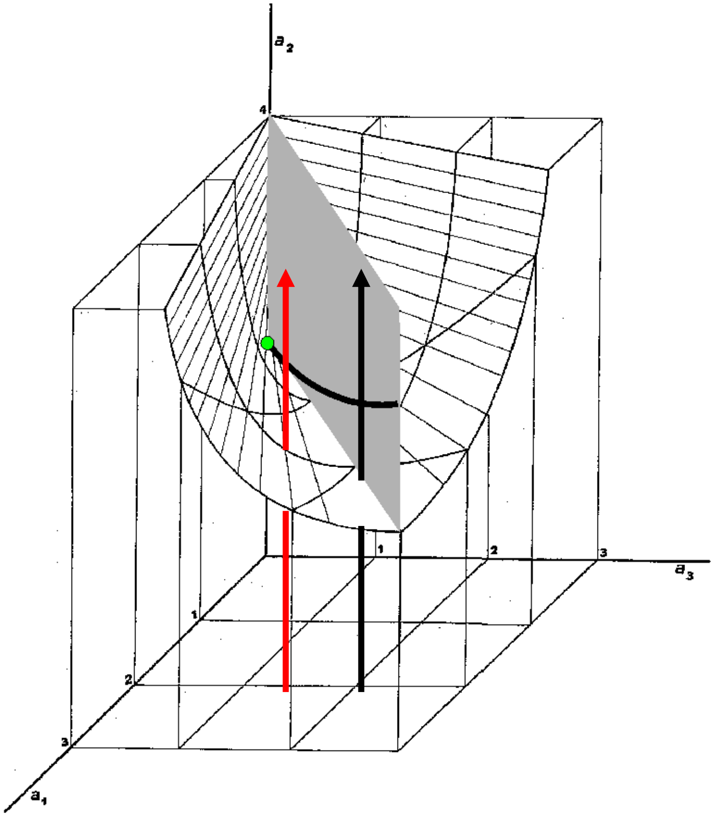


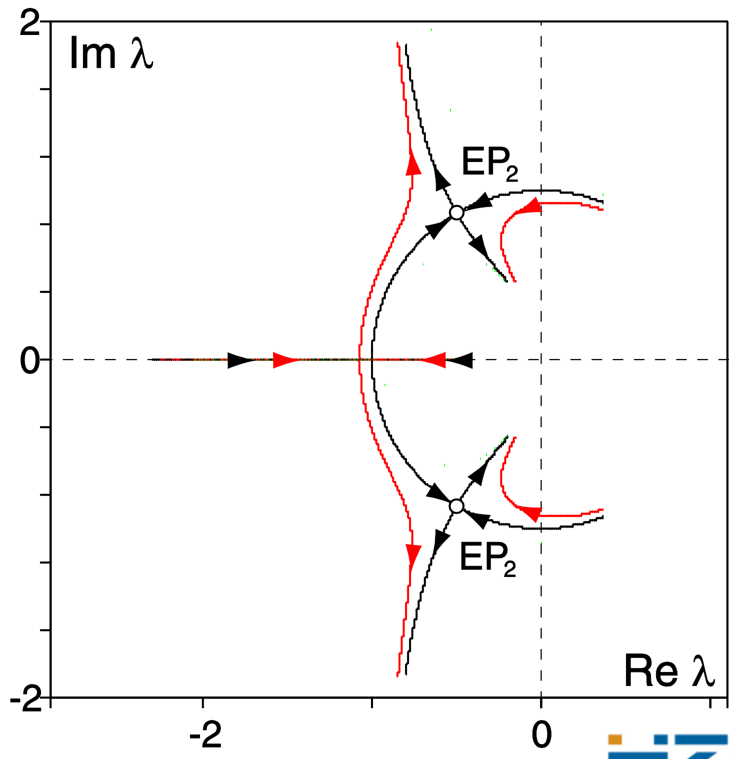
Exchange of instability between branches

Parameters change left to the tangent cone

$$a_1 = 2, \quad a_3 = 1.7, \quad 0 \le a_2 \le 6$$

Avoided crossings near EP_2, $\mathrm{Re}\lambda < 0$





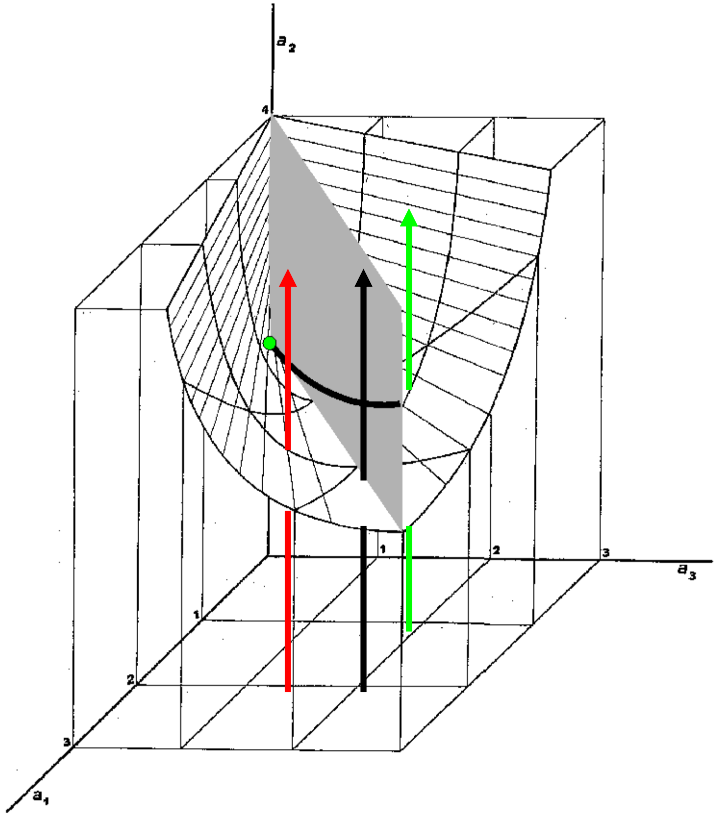
Oleg N. Kirillov, http://www.onkirillov.com

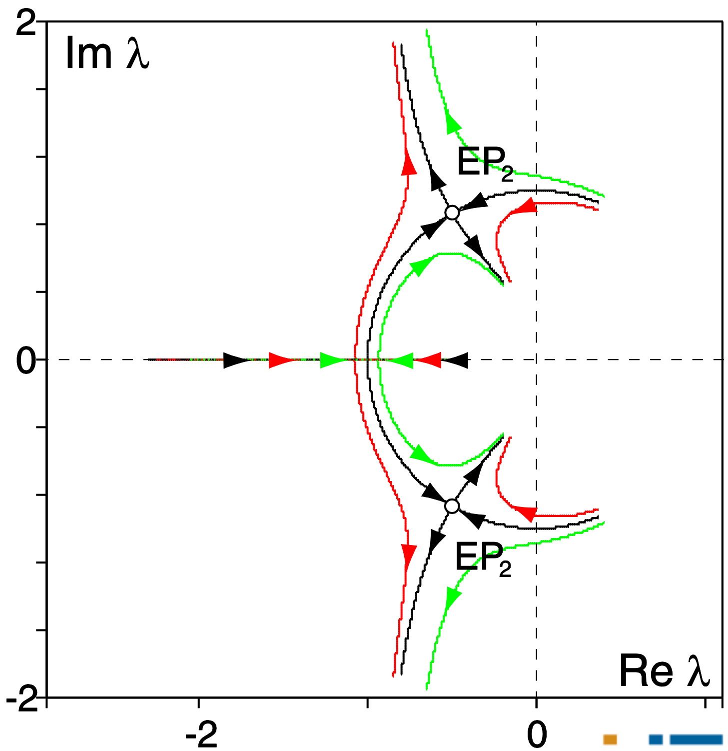
Selective role of the tangent cone

It determines which mode is destabilized by dissipation

because

the set of multiple complex eigenvalues (EP-set) is within it





Oleg N. Kirillov, http://www.onkirillov.com

Approximation of the eigenvalue trajectories

$$p = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1$$

A double eigenvalue at the EP-set: $a_{1,0} = a_{3,0}, \quad a_{2,0} = 2 + \frac{a_{1,0}^2}{4}$

$$\lambda_0 = -\frac{a_{1,0}}{4} + \frac{i}{4}\sqrt{16 - a_{1,0}^2}$$

Perturbation of parameters:

$$a_1 = a_{1,0} + \varepsilon\hat{a}_1, \quad a_2 = a_{2,0} + \varepsilon\hat{a}_2, \quad a_3 = a_{3,0} + \varepsilon\hat{a}_3$$

Newton-Puiseux series: $\lambda = \lambda_0 + \varepsilon^{1/2}\lambda_1 + \varepsilon\lambda_2 + \dots$

$$\lambda_1^2 = -2(\hat{a}_1\partial_{a_1}p + \hat{a}_2\partial_{a_2}p + \hat{a}_3\partial_{a_3}p)(\partial_\lambda^2 p)^{-1}$$

J. MORO et al. On the Lidskii-Vishik-Lyusternik perturbation theory for eigenvalues of matrices with arbitrary Jordan structure SIAM J. Matrix Anal. Appl. 18 (1997) 793-817

Approximation of the eigenvalue trajectories

Rewrite the perturbation formula to emphasize multiple parameters:

$$\mathrm{Re}\Delta\lambda + i\mathrm{Im}\Delta\lambda = \pm\sqrt{\langle \mathbf{f}, \Delta\mathbf{a}\rangle + i\langle \mathbf{g}, \Delta\mathbf{a}\rangle} + O(\|\Delta\mathbf{a}\|)$$

$$\Delta\lambda = \lambda - \lambda_0 \qquad \Delta\mathbf{a} = \varepsilon(\hat{a}_1, \hat{a}_2, \hat{a}_3)$$

$$\mathbf{f} = (f_1, f_2, f_3) \qquad \mathbf{g} = (g_1, g_2, g_3)$$

$$\gamma := (f_1 g_2 - f_2 g_1)\Delta a_1 + (f_3 g_2 - f_2 g_3)\Delta a_3$$

Compute the coefficients at the EP-set:

$$f_2 = \frac{8 - a_{1,0}^2}{2(a_{1,0}^2 - 16)}, \quad g_2 = \frac{a_{1,0}\sqrt{16 - a_{1,0}^2}}{2(a_{1,0}^2 - 16)}, \quad \gamma = \frac{4\sqrt{16 - a_{1,0}^2}}{(16 - a_{1,0}^2)^2}(\Delta a_3 - \Delta a_1)$$

HZDR

Approximation of the eigenvalue trajectories

Trajectories of eigenvalues in the complex plane
near an EP_2 point as a_2 varies:

$$g_2(\mathrm{Re}\Delta\lambda)^2 - 2f_2\mathrm{Re}\Delta\lambda\mathrm{Im}\Delta\lambda - g_2(\mathrm{Im}\Delta\lambda)^2 = \frac{4\sqrt{16-a_{1,0}^2}}{(16-a_{1,0}^2)^2}(\Delta a_3 - \Delta a_1)$$

If $\Delta a_3 = \Delta a_1$, i.e. perturbation is within the tangent cone,
then the eigenvalues merge at the 1:1 resonance

If $\Delta a_3 \neq \Delta a_1$, i.e. perturbation is outside the tangent cone,
then eigenvalues move along the branches of a hyperbola
and demonstrate an imperfect merging

Oleg KIRILLOV Nonconservative stability problems of modern physics
De Gruyter, Berlin, Boston, 2013, 429 pp.

Stability boundary via perturbation of eigenvalues

$$p = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1$$

In the ‚ideal' case $(a_1 = a_3 = 0):\quad \lambda_{id}^2 = -\frac{a_2}{2} \pm \frac{1}{2}\sqrt{a_2^2 - 4}$

Perturbation of a simple imaginary eigenvalue at $a_2 > 0$

$$\lambda = \lambda_{id} - a_1 \frac{\partial_{a_1} p}{\partial_\lambda p} - a_3 \frac{\partial_{a_3} p}{\partial_\lambda p} + o(a_1, a_3)$$

Approximating stability boundary by vanishing the linear increment:

$$a_1 \lambda_{id}^2(a_2) + a_3 = 0$$

The approximate boundary yields exactly the Hurwitz threshold:

$$a_2 = 2 + \frac{(a_1 - a_3)^2}{a_1 a_3}, \qquad \lim_{\varepsilon \to 0} \frac{(\varepsilon\hat{a}_1 - \varepsilon\hat{a}_3)^2}{\varepsilon\hat{a}_1 \varepsilon\hat{a}_3} - 2 = O(1)$$

Same ideas work in a more complicated case

Gyroscopic stabilization of non-conservative systems

$$\ddot{x} + (\gamma G + \delta D)\dot{x} + (\nu N + P)x = 0, \quad x \in R^m, \quad m \; is \; even$$

$$G = -G^T, \quad D = D^T, \quad N = -N^T, \quad P = P^T = -K < 0$$

Statically unstable potential system $\ddot{x} + Px = 0$

can be gyroscopically stabilized, i.e. $\ddot{x} + \gamma G\dot{x} + Px = 0$

is stable if $\det G \neq 0$ and $|\gamma| > \gamma_0 > 0$

Do the non-conservative perturbations δD and νN preserve the gyroscopic stabilization?

Oleg N. Kirillov, http://www.onkirillov.com

Same ideas work in a more complicated case

A double eigenvalue with the Jordan chain at the threshold of gyroscopic stabilization

$$(-I\omega_0^2 + i\omega_0\gamma_0 G - K)u_0 = 0$$

$$(-I\omega_0^2 + i\omega_0\gamma_0 G - K)u_1 = -(2i\omega_0 I + \gamma_0 G)u_0$$

Orthogonality
: $u_0^*(2i\omega_0 I + \gamma_0 G)u_0 = 0$

Rayleigh quotient: $\omega_0^2 = \dfrac{u_0^* K u_0}{u_0^* u_0}$

Hamiltonian-Hopf bifurcation

$$i\omega(\gamma) = i\omega_0 \pm i\mu\sqrt{\gamma - \gamma_0} + O(\gamma - \gamma_0)$$

$$u(\gamma) = u_0 \pm i\mu u_1\sqrt{\gamma - \gamma_0} + O(\gamma - \gamma_0)$$

$$\mu^2 = -\frac{2\omega_0^2 u_0^* u_0}{\gamma_0(\omega_0^2 u_1^* u_1 + u_1^* K u_1 - i\omega_0\gamma_0 u_1^* G u_1 - u_0^* u_0)}$$

Same ideas work in a more complicated case

Non-conservative perturbation of simple eigenvalues at $\gamma > \gamma_0$

$$\lambda = i\omega + \frac{\omega^2 u^* D u \delta - i\omega u^* N u \nu}{u^* K u - \omega^2 u^* u} + o(\delta, \nu)$$

Stability boundary: $$\nu = \beta(\gamma)\delta, \quad \beta(\gamma) = -i\omega(\gamma)\frac{u^*(\gamma) D u(\gamma)}{u^*(\gamma) N u(\gamma)}$$

New threshold of gyroscopic stability: $$\gamma_{cr}(\beta) = \gamma_0 + \frac{n_1^2(\beta - \beta_0)^2}{\mu^2(\omega_0 d_2 - \beta_0 n_2 - d_1)^2}$$

Valid at: $$|\beta - \beta_0| \ll 1, \quad \beta_0 = \beta(\gamma_0) = -i\omega_0 \frac{u_0^* D u_0}{u_0^* N u_0}$$

$$d_1 = \mathrm{Re}(u_0^* D u_0), \quad d_2 = \mathrm{Im}(u_0^* D u_1 - u_1^* D u_0)$$

$$n_1 = \mathrm{Im}(u_0^* N u_0), \quad n_2 = \mathrm{Re}(u_0^* N u_1 - u_1^* N u_0)$$

Gyroscopic stabilization in case of m=2

$$\ddot{\mathbf{z}} + (\delta\mathbf{D} + \Omega\mathbf{J})\dot{\mathbf{z}} + (\mathbf{K} + \nu\mathbf{J})\mathbf{z} = 0,$$

$$\mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{D} = \mathbf{D}^T, \quad \mathbf{K} = \mathbf{K}^T,$$ κ_1, κ_2 eigenvalues of $\mathbf{K}$

$\delta=0$, $\nu=0$, eigenvalues: $\lambda = i\omega_{\pm}(\Omega)$

$$\omega_{\pm}(\Omega) = \sqrt{\omega_0^2 + \frac{\Omega_2}{2}\left(\sqrt{\Omega^2 - \Omega_2^2} \pm \sqrt{\Omega^2 - \Omega_1^2}\right)\sqrt{\frac{\Omega^2}{\Omega_2^2} - 1}}$$

$$\omega_0 = \frac{1}{2}\sqrt{\Omega_2^2 - \Omega_1^2}$$

$$0 \le \sqrt{-\kappa_2} - \sqrt{-\kappa_1} =: \Omega_1 \le \Omega_2 := \sqrt{-\kappa_1} + \sqrt{-\kappa_2}$$

Gyroscopic stabilization in case of **m=2**

$$\mathbf{K} = \begin{pmatrix} -1 & 0 \\ 0 & -4 \end{pmatrix}$$

$\delta=0$, $\nu=0$: When Ω increases, complex eigenvalues $\lambda = i\omega_{\pm}(\Omega)$

move along the circle in the complex plane

$$(\mathrm{Re}\lambda)^2 + (\mathrm{Im}\lambda)^2 = \omega_0^2$$

After the Krein collision at $\Omega=\Omega_2$,
pure imaginary eigenvalues diverge along the imaginary axis

$$\omega_+(\Omega) > \omega_-(\Omega) > 0$$

Krein signature is positive for $i\omega_+(\Omega)$, negative for $i\omega_-(\Omega)$

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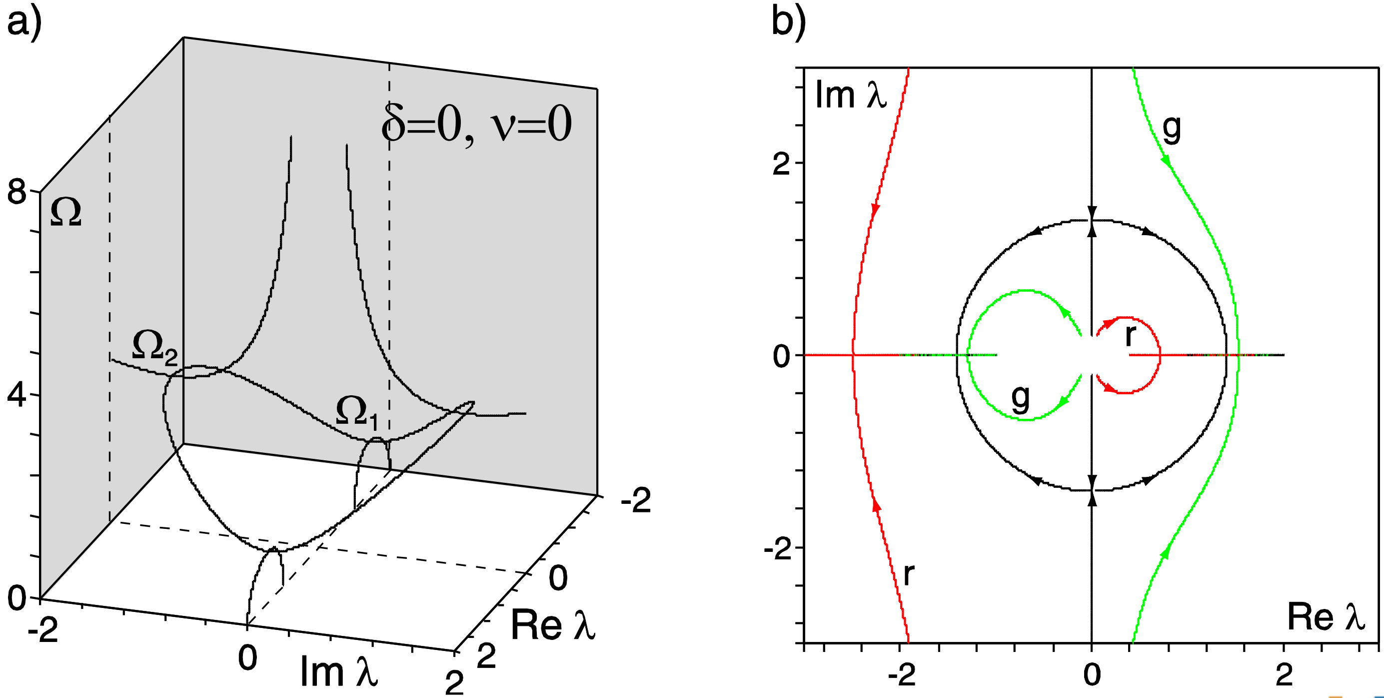
Full dissipation (δ=1, ν=0) destabilizes eigenvalues with negative Krein signature (red curves)

Circulatory forces (δ=0, ν=1) destabilize eigenvalues with positive Krein signature (green curves)

$$\mathbf{D} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} > 0$$

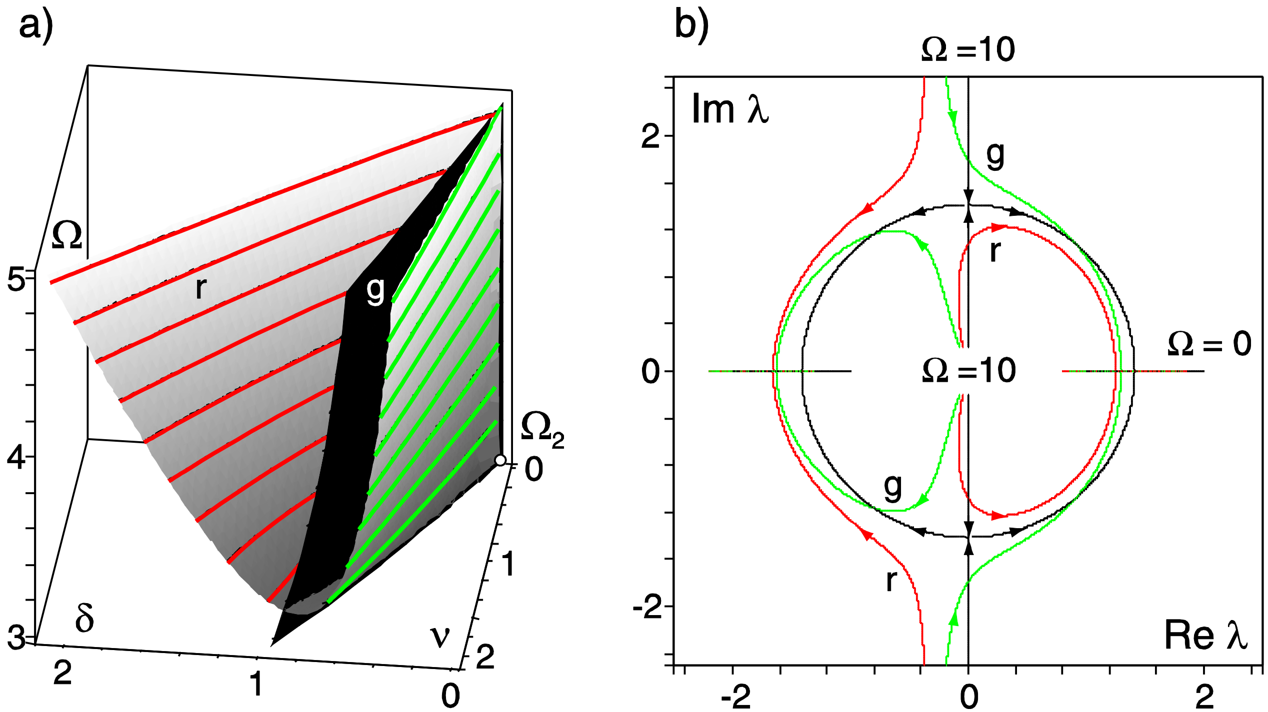


Gyroscopic stabilization

in the presence of damping and non-conservative positional forces

$$\kappa_1 = -1, \quad \kappa_2 = -4, \quad \mathrm{tr}\mathbf{D} = 3, \quad \mathrm{tr}\mathbf{KD} = -6, \quad \det \mathbf{D} = 1$$

$\delta = 0.3, \nu = 0.6$ red eigencurves $\delta = 0.3, \nu = 0.9$ green eigencurves



Switching surface

has a tangent cone as its linear approximation at the singular point

$$\nu = \delta\Omega \frac{\delta^2 \mathrm{tr}\mathbf{D} \det \mathbf{D} + 4\Omega_2\gamma_* + \mathrm{tr}\mathbf{D}(\Omega^2 - \Omega_2^2)}{\delta^2(\mathrm{tr}\mathbf{D})^2 + 4\Omega^2}$$

Tangent cone:

$$\{\nu = \gamma_*\delta, \quad \Omega > \Omega_2, \quad \nu > 0, \quad \delta > 0\} \quad \gamma_* = \frac{\mathrm{tr}\mathbf{KD} + \left(\Omega_2^2 - \omega_0^2\right)\mathrm{tr}\mathbf{D}}{2\Omega_2}$$

Movement of eigenvalues (approximation near singularity):

$$(\mathrm{Im}\lambda - \omega_0 - \mathrm{Re}\lambda - a/2)^2 - (\mathrm{Im}\lambda - \omega_0 + \mathrm{Re}\lambda + a/2)^2 = 2d$$

$$a = \frac{\mathrm{tr}\mathbf{D}}{2}\delta, \quad d = \frac{\Omega_2}{2\omega_0}(\gamma_*\delta - \nu)$$

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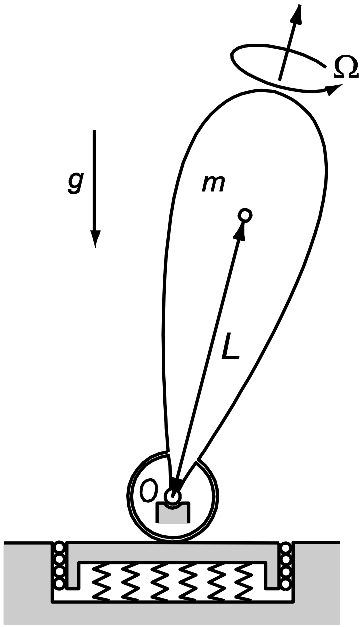
1995 Crandall

A gyropendulum with stationary and rotating damping

$$\ddot{\mathbf{z}} + \begin{pmatrix} \sigma+\rho & \eta\Omega \\ -\eta\Omega & \sigma+\rho \end{pmatrix} \dot{\mathbf{z}} + \begin{pmatrix} -\alpha^2 & \rho\Omega \\ -\rho\Omega & -\alpha^2 \end{pmatrix} \mathbf{z} = 0$$

$$\eta = \frac{I_a}{I_d}, \quad \sigma = \frac{b_s}{I_d}, \quad \rho = \frac{b_r}{I_d}, \quad \alpha^2 = \frac{mgL}{I_d}$$



Drag force,
stationary damping (b_s)

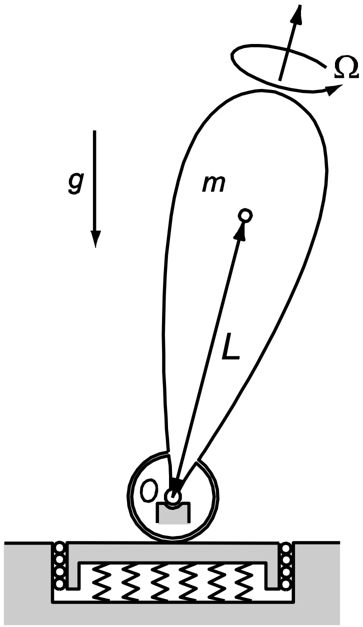
„Viscous friction force“,
rotating damping (b_r)

1995 Crandall

A gyropendulum with stationary and rotating damping

$$\ddot{\mathbf{z}} + \begin{pmatrix} \sigma+\rho & \eta\Omega \\ -\eta\Omega & \sigma+\rho \end{pmatrix} \dot{\mathbf{z}} + \begin{pmatrix} -\alpha^2 & \rho\Omega \\ -\rho\Omega & -\alpha^2 \end{pmatrix} \mathbf{z} = 0$$

$$\eta = \frac{I_a}{I_d}, \quad \sigma = \frac{b_s}{I_d}, \quad \rho = \frac{b_r}{I_d}, \quad \alpha^2 = \frac{mgL}{I_d}$$



Gyroscopic stabilization (σ,ρ=0):

$$\Omega > \Omega_0^+ = \frac{2\alpha}{\eta}$$

Asymptotic stability (σ,ρ≠0):

$$\Omega^2 > {\Omega_0^+}^2 + \frac{1}{\rho}\frac{\alpha^2}{\eta^2}\frac{(\sigma\eta+\rho(\eta-2))^2}{\sigma\eta+\rho(\eta-1)} \geq {\Omega_0^+}^2$$

$$\sigma + \rho > 0$$

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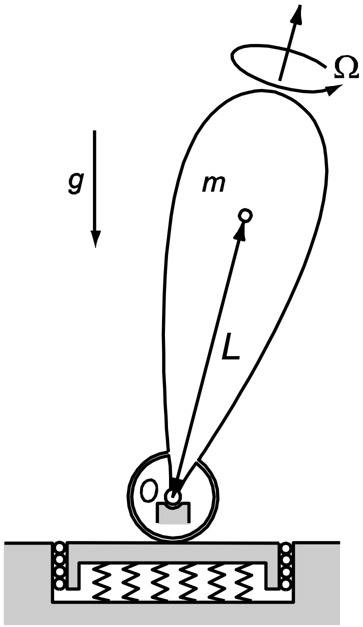
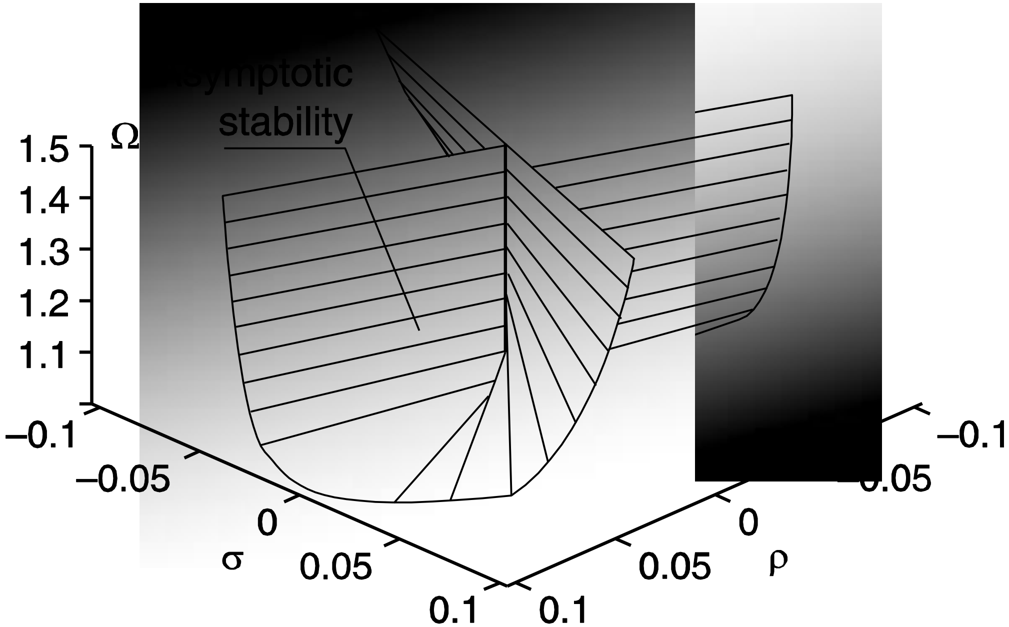
1995 Crandall

A gyropendulum with stationary and rotating damping

$$\ddot{\mathbf{z}} + \begin{pmatrix} \sigma+\rho & \eta\Omega \\ -\eta\Omega & \sigma+\rho \end{pmatrix}\dot{\mathbf{z}} + \begin{pmatrix} -\alpha^2 & \rho\Omega \\ -\rho\Omega & -\alpha^2 \end{pmatrix}\mathbf{z} = 0$$

$$\eta = \frac{I_a}{I_d}, \quad \sigma = \frac{b_s}{I_d}, \quad \rho = \frac{b_r}{I_d}, \quad \alpha^2 = \frac{mgL}{I_d}$$





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2008 Samantaray et al.

Fast/slow precession destabilization of the Crandall gyropendulum

Fast: $\dfrac{\sigma}{\rho} < \dfrac{2-\eta}{\eta}$

Slow: $\dfrac{\sigma}{\rho} > \dfrac{2-\eta}{\eta}$

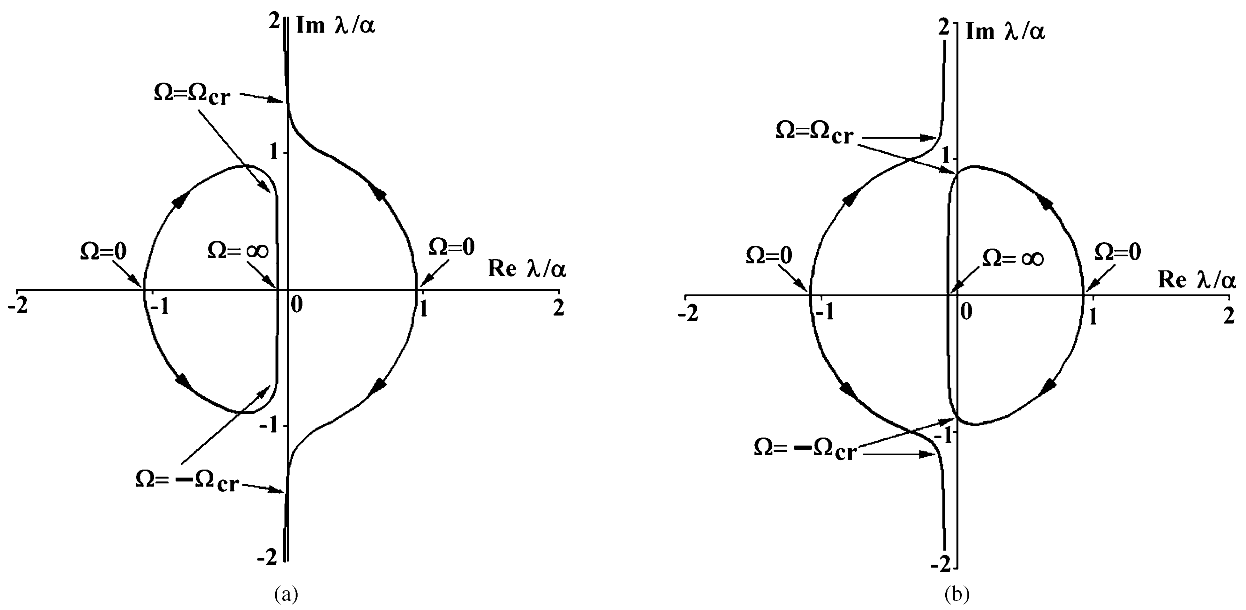


Fig. 2. Variation of the eigenvalues (scaled) of the gyropendulum. (a) Case $\sigma/\rho < (2-\eta)/\eta$ with parameter values $\eta = 1.5$, $\alpha = 10\ \mathrm{s}^{-1}$, $\sigma = 0$ and $\rho = 1\ \mathrm{s}^{-1}$. (b) Case $\sigma/\rho > (2-\eta)/\eta$ with parameter values $\eta = 1.5$, $\alpha = 10\ \mathrm{s}^{-1}$, $\sigma = 0.5\ \mathrm{s}^{-1}$ and $\rho = 1\ \mathrm{s}^{-1}$.

2008 Samantaray et al.

Fast/slow precession destabilization of the Crandall gyropendulum

Fast: $\frac{\sigma}{\rho} < \frac{2-\eta}{\eta}$ Slow: $\frac{\sigma}{\rho} > \frac{2-\eta}{\eta}$

The threshold of gyroscopic stabilization

$$\Omega^2 = {\Omega_0^+}^2 + \frac{1}{\rho}\frac{\alpha^2}{\eta^2}\frac{\boxed{(\sigma\eta + \rho(\eta-2))}^2}{\sigma\eta + \rho(\eta-1)}$$

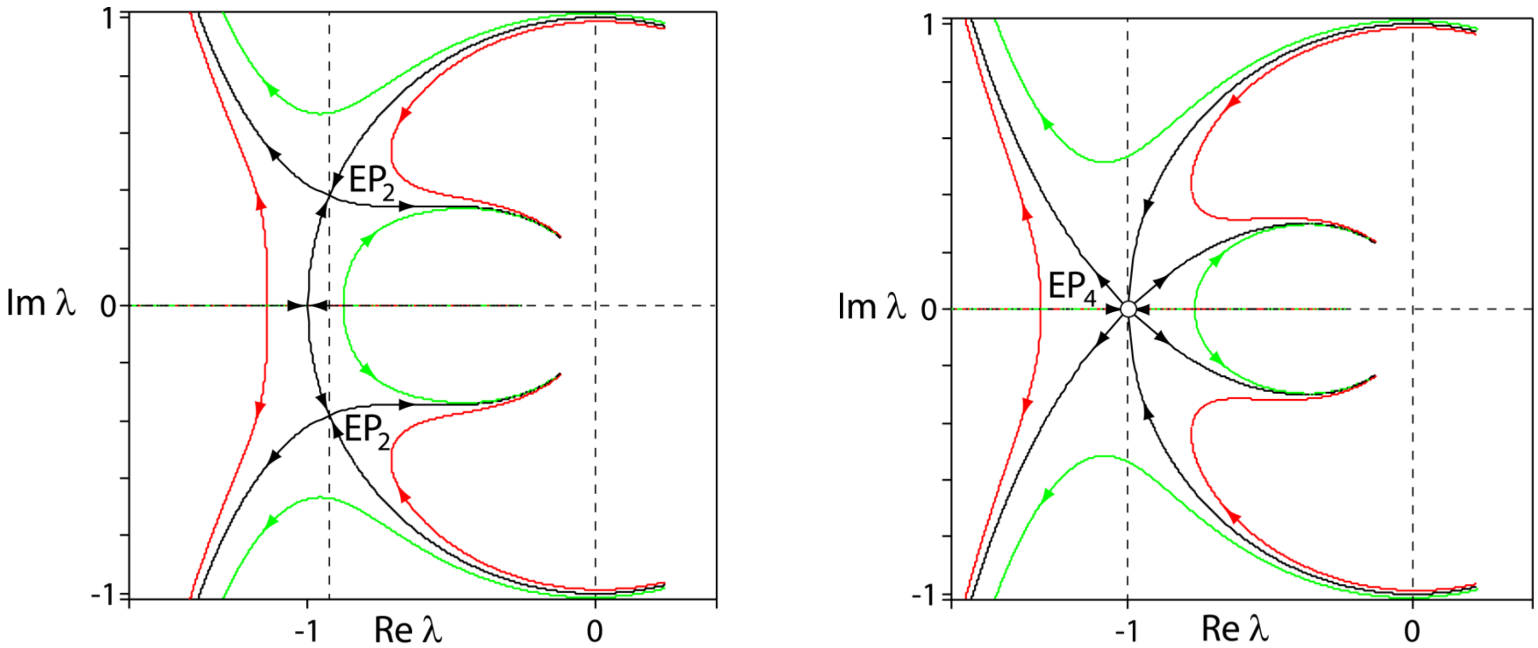
The tangent cone: $\boxed{\frac{\sigma}{\rho} = \frac{2-\eta}{\eta},}$ $\Omega > \Omega_0^+, \quad \rho > 0$

selects whether the fast or slow precession is destabilized

Robust stabilizaton by minimizing the root abscissa

$$p = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1 \qquad \mathbf{a}(p) = \max\{\mathrm{Re}\lambda : p(\lambda) = 0\}$$

$$\mathbf{a}^* = \inf_{a_1, a_2, a_3} \mathbf{a}(p)$$



The minimizer seems to be at the EP-set

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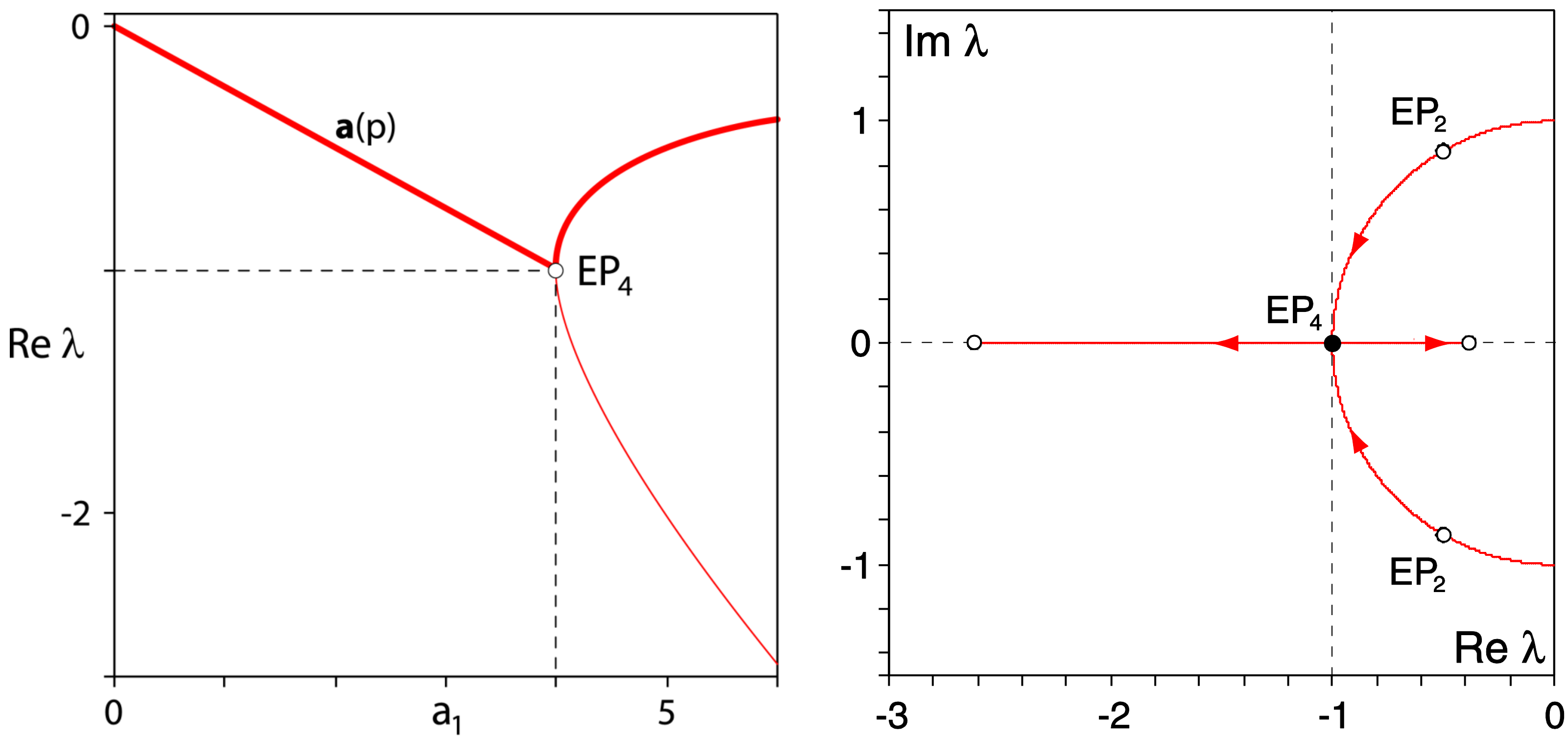
Robust stabilizaton by minimizing the root abscissa

Multiple eigenvalues at the EP-set

$$\lambda_1 = \lambda_2 = -\frac{a_1}{4} - \frac{1}{4}\sqrt{a_1^2 - 16} \qquad \lambda_3 = \lambda_4 = -\frac{a_1}{4} + \frac{1}{4}\sqrt{a_1^2 - 16}$$



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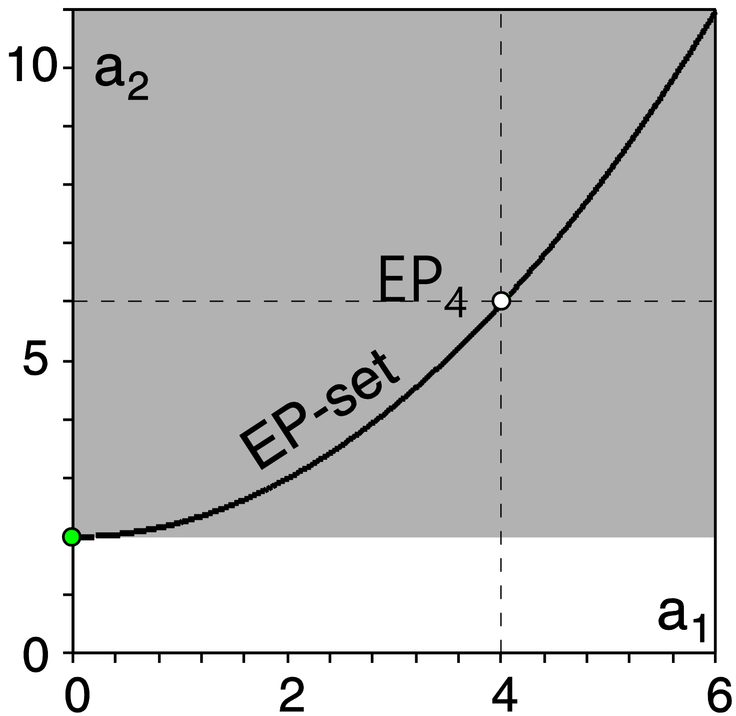
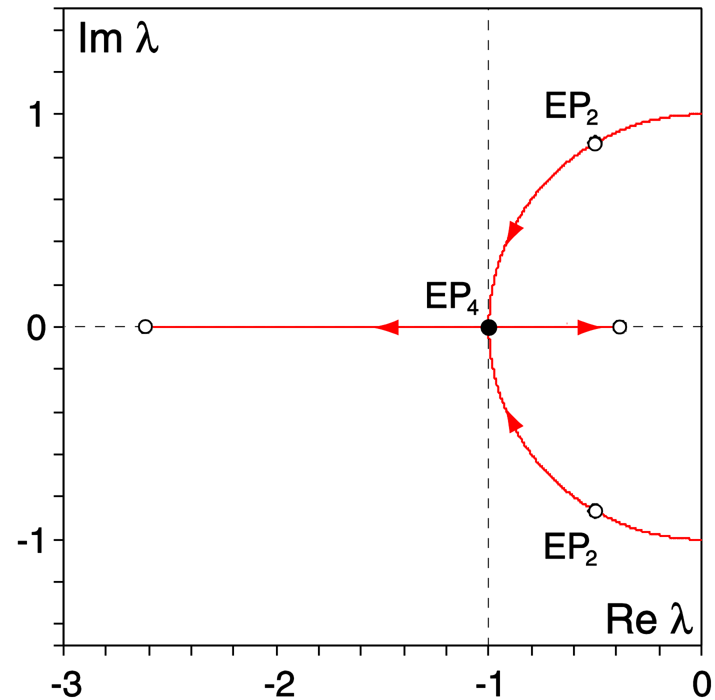
Robust stabilizaton by minimizing the root abscissa

$$\mathrm{EP}_4: \quad \lambda = -1, \quad a_1 = 4, \quad a_2 = 6, \quad a_3 = 4$$

The minimizer $p^*(\lambda) = \lambda^4 + 4\lambda^3 + 6\lambda^2 + 4\lambda + 1 = (\lambda + 1)^4$





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Robust stabilizaton by minimizing the root abscissa

Th. (Blondel et al. 2012): Given a family of real polymomials and a single affine constraint

$$P = \{\lambda^n + a_1\lambda^{n-1} + \ldots + a_{n-1}\lambda + a_n : \quad b_0 + \sum_{j=1}^{n} b_j a_j = 0\}$$

Let $h(\lambda) = b_n\lambda^n + b_{n-1}C_n^{n-1}\lambda^{n-1} + \ldots + b_1 C_n^1\lambda + b_0$

$$k = \max\{j : \ b_j \neq 0\}$$

Then

$$\mathbf{a}^* := \inf_{p\in P} \mathbf{a}(p) = -\max\{\zeta \in R : h^{(i)}(\zeta) = 0 \ \ for\ some \ \ i \in \{0, \ldots, k-1\}\}$$

The minimizing polynomial $p^*(\lambda) = (\lambda - \gamma)^n \in P, \quad \gamma = \mathbf{a}^*$

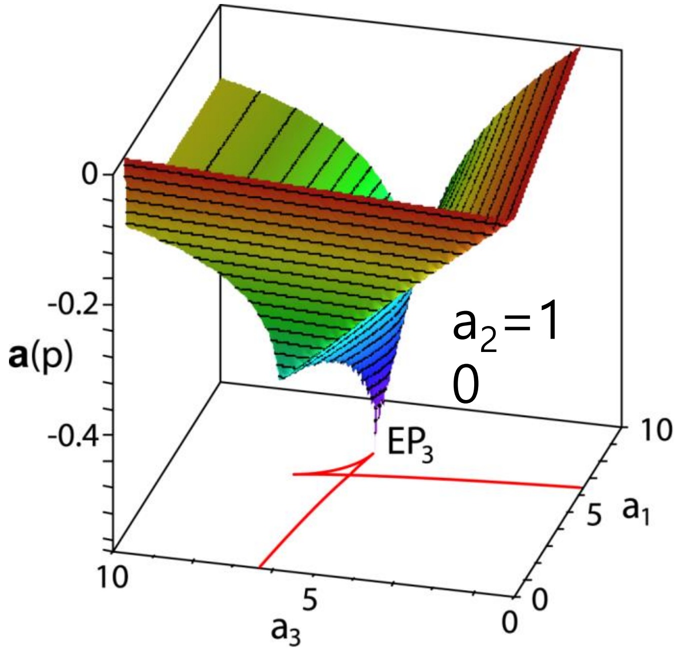
if and only if -$\mathbf{a}^*$ is a root of *h*.

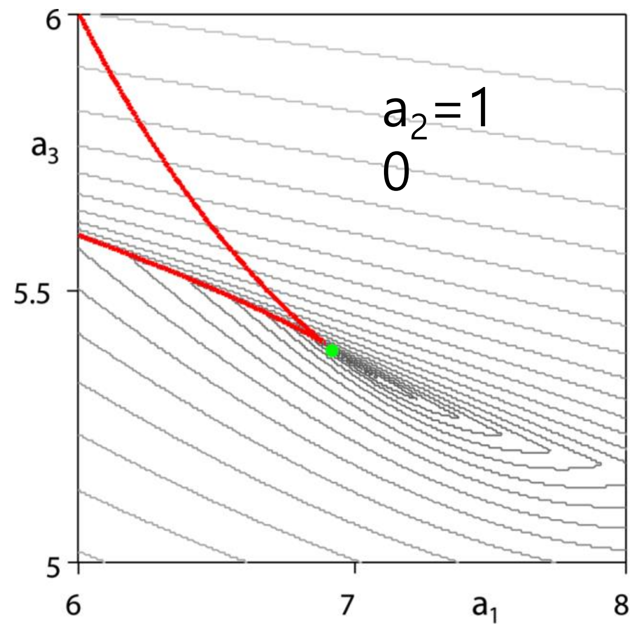
V. D. BLONDEL et al. Explicit solutions for root optimization of a polynomial family with one affine constraint IEEE Trans. Autom. Control 57 (2012) 3078–89.

Robust stabilizaton by minimizing the root abscissa

$$p = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1$$

Additional constraint (a_2=const) reduces the root multiplicity from 4 to 3 at the minimum





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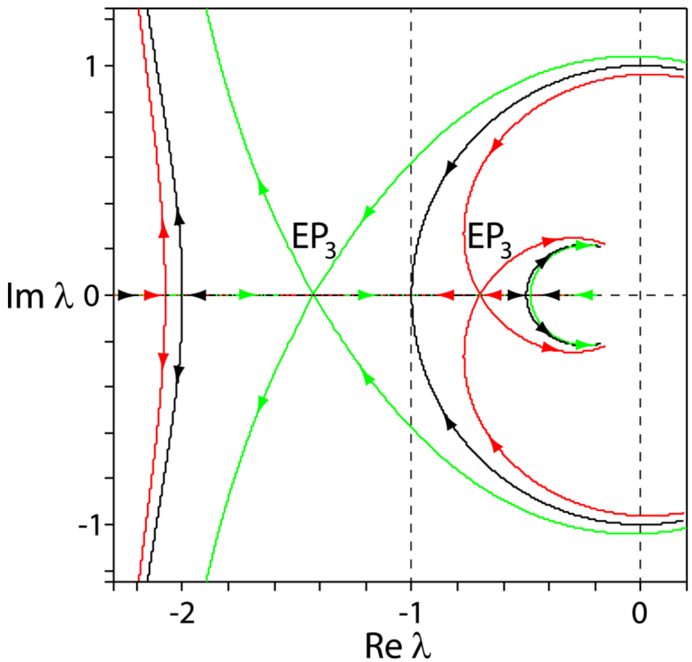
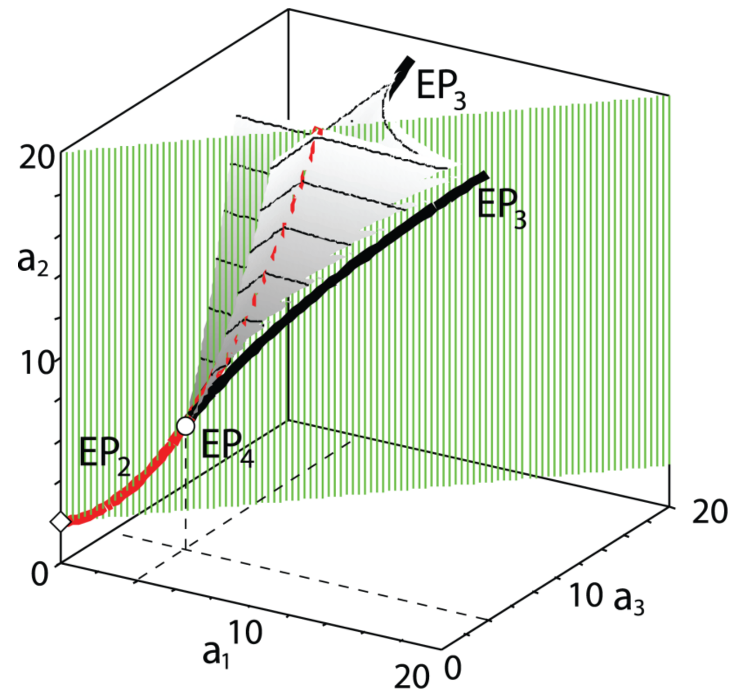
Minima of the root abscissa and discriminant

$$p = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + 1$$

Discriminant of the polynomail has a Swallowtail singularity at EP_4 and cuspidal edges at EP_3





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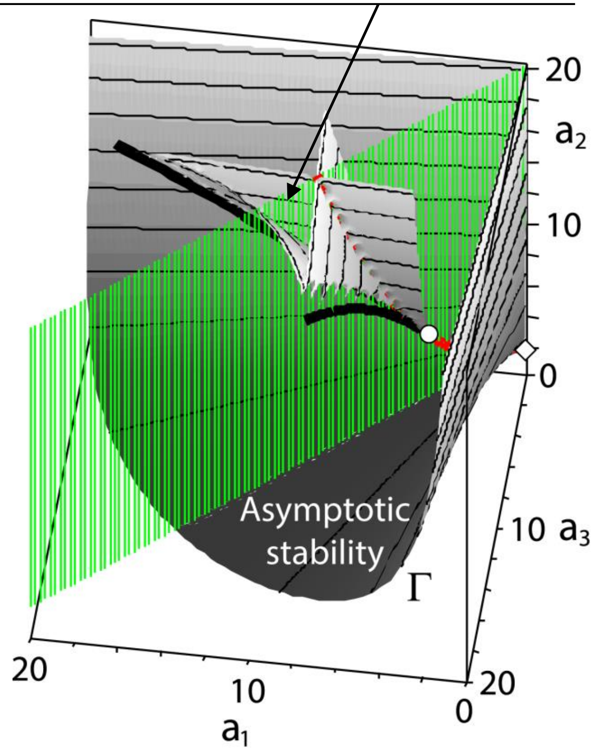
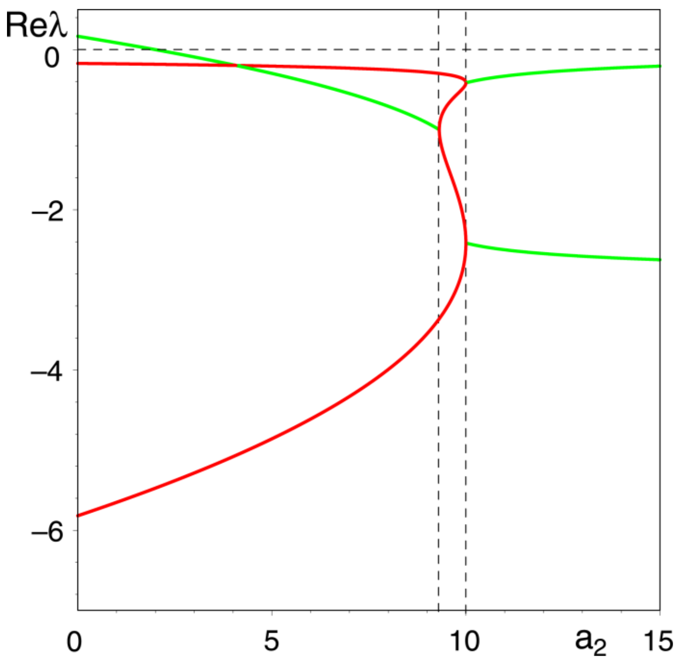
Asymptotic stability and heavy damping

All the roots are real and simple inside the „spike“ (heavy damping).

Root abscissa minimization yields the points at the boundary of the domain of heavy damping.





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Asymptotic stability and heavy damping

$$\ddot{x} + D\dot{x} + Kx = 0, \quad x \in R^m$$

In a heavily damped system all the eigenvalue of $L(\lambda) = \lambda^2 + D\lambda + K$ are semi-simple real and negative

Spectral abscissa of the matrix polynomial $L(\lambda)$

$$\alpha(D,K) = \mathbf{a}(\det(L(\lambda))) = \max\{\mathrm{Re}\lambda : \quad \det(L(\lambda)) = 0\}$$

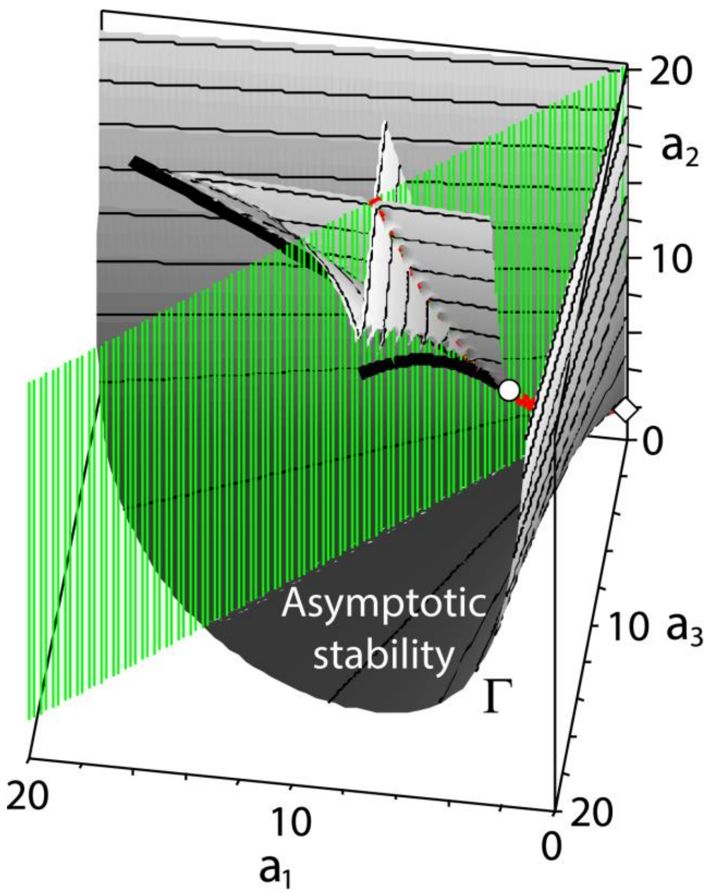
$$\alpha^* = \inf_D \alpha(D,K)$$

Freitas and Lancaster (1999) $\forall K = K^T > 0 : \quad \alpha^* \geq -\sqrt[2m]{\det K};$

$\alpha^* = -\sqrt[2m]{\det K}$ iff $L(\lambda)$ has an eigenvalue of multiplicity 2m

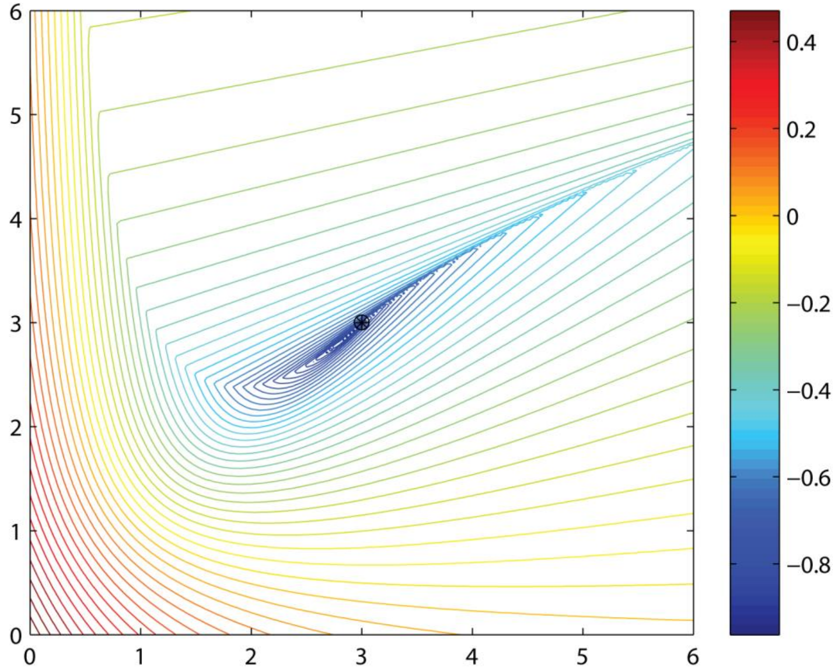
P. FREITAS, P. LANCASTER On the optimal value of the spectral abscissa for a system of linear oscillators SIAM J Matrix Anal. Appl. 21 (1999) 195–208

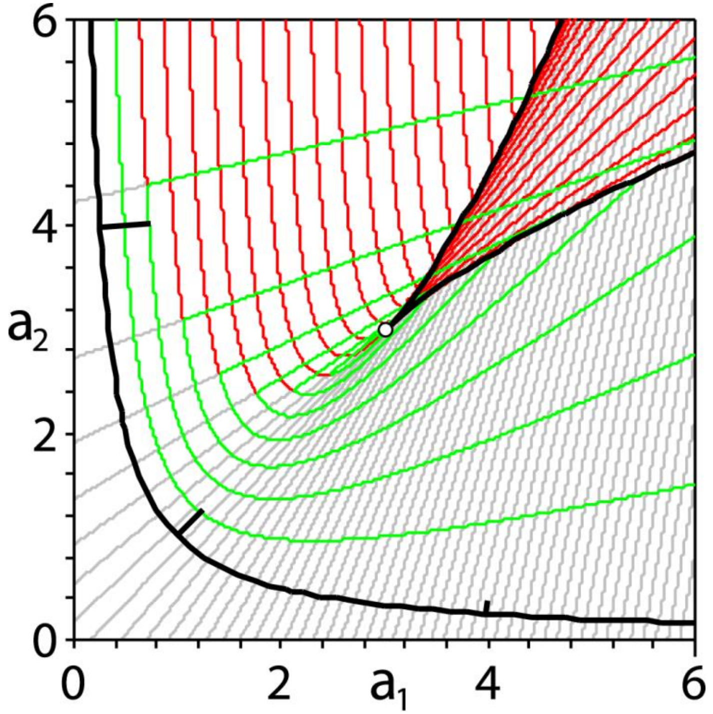
Robust stability at the Swallowtail



1. Multiple eigenvalues mysteriously originate in various contexts of optimizing stability
2. They are related to singularities on the stability boundary and the discriminant set
3. Why optimization of abscissa so naturally tends to the most degenerate eigenvalue?

Does optimizing the abscissa possesses a physical interpretation in terms of wave front propagation with multiple roots corresponding to caustics?





$$p=\lambda^3+a_1\lambda^2+a_2\lambda+1$$

V. I. ARNOLD Lectures on bifurcations in versal families Russ. Math. Surv. 27 (1972) 54–123

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NONCONSERVATIVE STABILITY PROBLEMS OF MODERN PHYSICS

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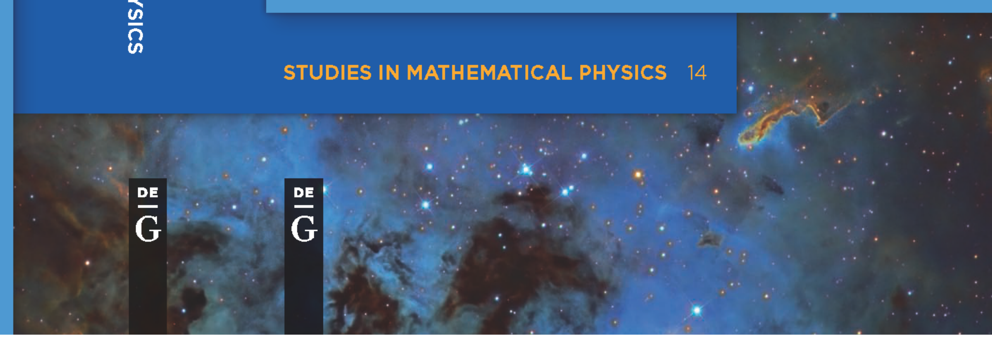

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This work gives a complete overview on the subject of nonconservative stability from the modern point of view. Relevant mathematical concepts are presented, as well as rigorous stability results and numerous classical and contemporary examples from mechanics and physics.

It deals with both finite- and infinite-dimensional nonconservative systems and covers the fundamentals of the theory, including such topics as Lyapunov stability and linear stability analysis, Hamiltonian and gyroscopic systems, reversible and circulatory systems, influence of structure of forces on stability, and dissipation-induced instabilities, as well as concrete physical problems, including perturbative techniques for nonself-adjoint boundary eigenvalue problems, theory of the destabilization paradox due to small damping in continuous circulatory systems, Krein-space related perturbation theory for the MHD kinematic mean field α^2-dynamo, analysis of Campbell diagrams and friction-induced flutter in gyroscopic continua, non-Hermitian perturbation of Hermitian matrices with applications to optics, and magnetorotational instability and the Velikhov-Chandrasekhar paradox.

The book serves present and prospective specialists providing the current state of knowledge in the actively developing field of nonconservative stability theory. Its understanding is vital for many areas of technology, ranging from such traditional ones as rotor dynamics, aeroelasticity and structural mechanics to modern problems of hydro- and magnetohydrodynamics and celestial mechanics.

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